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The Potential of Covered Profiled Steel Cladding as a Building-Integrated Solar Collector for the UK Climate

by

Kam Ting Kenneth Ho

A Doctoral Thesis

Submitted in partial fulfillment of the requirements for the award of

Doctor of Philosophy of Loughborough University

September 1998

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Acknowledgments

I praise my Lord God Saviour Jesus Christ for His grace and unfailing love, compassion and kindness, for sustaining me throughout the research and, most of all, for His atoning sacrifice for the sins of many who come to Him.

I would like to express my gratitude to my parents for their love and financial support throughout my stay at Loughborough, and for their encouragement in times of distress.

I am greatly indebted to my supervisor, Dr Dennis L. Loveday, for his supervision, ideas, guidance and encouragement throughout the research, and for his assistance in the compilation of this thesis.

I express my thanks to various staff at the Department of Civil and Building Engineering who helped during the research. Specially thanks go to:

- Mr Alistar Gibb for assistance on the technical aspects of profiled metal-clad buildings;
- Dr Jonathan Weight and Dr Phil Haves for assistance in the aspect of computer simulation;
- Mr Mick Barker and Mr Dean Sanham for the construction of the solar collector test rig and various technical supports.

The assistance from Mr Andrew Ng for the use of MATLAB in deriving the mathematical model, is gratefully acknowledged.

Special thanks go to Mr Ryan Lim and Miss Ruki Manicavasaga for their moral support and help, Dr Francis Edum-Fotwe for being a helpful and sensitive colleague, and my sister and my brother-in-law for their encouragement and love.
Summary

Profiled steel cladding can be modified to act as an air heating solar collector by the addition of a transparent cover system. A mathematical model of the thermal performance of such an arrangement has been derived for the situation of a building-integrated solar collector facade, allowing for the condition of differing temperatures at front and rear faces of the collector. By introducing an equivalent ambient temperature, it is possible to quantify the performance of such a collector arrangement in terms of existing parameters as derived in the standard Hottel-Whillier-Bliss analysis. Using a purpose-built solar simulator, a set of standard performance characteristics for the proposed collector geometry is derived; these characteristics are used to confirm the validity of the derived model for use in this application area, i.e. as a building-integrated system with the standard thickness of back insulation. Those conditions of front / rear temperature difference and rear insulation level for which the standard Hottel-Whillier-Bliss analysis is no longer valid, are identified.

The model has been encoded as a new subroutine within the thermal simulation program TRNSYS in order to investigate the energy performance of a typical profiled metal-clad building in the UK climate with and without the assistance of such a collector system. The effects of orientation of the solar-collector facade, together with collection area, steel-to-cover spacing and fan power requirements were determined. Assessment of capital maintenance, operating costs and energy savings permitted the cost-effectiveness of such a system to be evaluated. Guidance for future designers of such building-integrated systems is presented for UK conditions. It is concluded that the use of such a collector system can approach cost-effectiveness in electrically-fuelled buildings, and that this is likely to be especially so if the building has a significant requirement for pre-heated fresh air. The system is shown to be not cost-effective at present for gas-fuelled installation in the UK, such as in the case of a retrofit to a typically profiled-clad sports centre, though factors other than that of payback alone may well influence such investment decisions in the future.
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1.1. General

For a considerable time man has been dependent on energy sources such as coal, electricity and gas in everyday life. Towards the end of 19th century, coal- and steam-based industries had reached their peaks and novel sources of energy were at their beginnings. After the Second World War, oil was much cheaper than coal and most industries changed over from coal to oil, increasing very sharply the demand for oil. This changing pattern of fuel utilisation due to massive industrialisation in major industrialised countries has led to the shortage of fuel supplies commonly referred to as 'the energy crisis' (Garg, 1982).

The importance of energy in the world economy was made evident by the oil embargo of 1973, when oil prices rose dramatically. Since then, the proliferation of ideas on the development and utilization of alternative energy sources and their practical application have been of increasing importance.

As well as the finite quantities of non-renewable energy sources acting as a stimulant to research, other factors began to have an influence; several studies in the 1980's highlighted the growing view that the ecological consequences of fossil fuel-based energy consumption had to be taken seriously (Røstvik, 1992). By the end of 1990, clear signs of environmental damage, such as pollution, global warming and damage to the ozone layer, were emerging; this increased interest in the commitment to saving energy.

Greater awareness of the threatening climatic situation, coupled with the increased interest in renewable energy usage, in particular solar energy, as a means to help solve this problem, has raised hopes of a improving the world's future ecological development (Røstvik, 1992).

In almost all industrialised countries, solar energy research and its application comprises a significant fraction of all renewable energy research. This is understandable since all forms of renewable energy (tidal, wind, hydro-electric and solar), apart from geothermal energy, originate from the sun. The use of solar energy directly would thus appear to be the most logical and efficient approach. The utilisation of tidal, wind and hydro-electric sources
frequently requires large scale devices to convert them to useful energy, making such systems unfeasible for ordinary individuals. Individual householders living in an ordinary dwelling with a flat or pitched roof might benefit, however, from investment in solar energy collection.

There are a number of advantages to using solar energy:

- Since solar energy is easily 'captured', even in small scale (such as on the roofs of buildings), individual buildings can 'collect' the energy. Although large scale conversion of solar energy does exist, there are only a relatively small number of such installations in the world.
- There is less risk of disturbing the local environment than would be the case for other renewable energies such as tidal and hydro-electric power. The use of hydro-electric energy, for example, would alter the local water level and thus the natural environment of the surroundings.
- The capital cost of small scale solar energy capturing equipment is less than that for wind, tidal and hydro-electric energy capture schemes, thus allowing individual enterprise in its exploitation.

As regards the world's consumption of energy, it is the built environment that has traditionally been one of the major consumers. For a long time, solar energy has been investigated and exploited as a means for space and / or water heating in buildings. In those parts of the world where sunshine is abundant and reliable, solar water heating has been relatively cost-effective. For temperate climates such as the UK the main requirement in buildings is for space heating and for lighting. Although solar energy cannot usually meet the full space heating requirement of a building in a temperate climate zone such as the UK, it can certainly supply part of the heating load and can assist an installed conventional heating system. A system that operates in this way is referred to as a solar assisted heating system. Usually such a systems is used to heat air. However, the combined effects of reduced insulation in temperate zones together with high capital costs of solar equipment have resulted in poorer cost-effectiveness of solar heating as compared with the conventional heating methods that are available. There is thus an understandable reluctance to take the long-term energy benefits offered by a renewable energy system when the short-term economics seem unattractive (Jefferson, 1994).
One method of improving the cost-effectiveness of a solar air heating system is to employ the concept of 'multi-functionality'. Here, a component of a building fulfills more than one role, such as that of a weatherproof envelope combined with energy collection and, sometimes, storage. Both passive and active solar heating systems can employ this concept. Over the last few years, however, other developments have taken place which could substantially change the situation regarding solar heating in northern latitudes, together with the attitudes of building operators and investors. Such developments have been driven principally by the concerns over climate change, and are described in the next section.

1.2. New opportunities for solar assistance in buildings

Carbon dioxide is one of the main 'greenhouse' gases, and energy use in buildings contributes about 50% to the carbon dioxide emissions of industrialised countries. In this respect, air conditioning has been recognised as an energy-intensive means for tempering a building's internal environment. This has led to a trend towards low energy architecture, which utilises good insulation, thermal mass, and, where possible, natural ventilation (Bunn, 1997). In deeper plan buildings, it is necessary to introduce this air either by mechanical ventilation, or by ventilation induced by natural forces such as the stack effect. This trend towards 'greener' buildings and 'sustainable cities' is certain to continue for the foreseeable future. Many future new buildings are likely to exploit these techniques, and there exists the possibility of integrating air-heating solar collection into such systems where appropriate.

The performance of photovoltaic cells has steadily improved, resulting in an efficiency of up to 17% for the conversion of solar radiation into electricity. This, together with the de-regulation of national electricity supplies, is progressively increasing the cost-effectiveness of photovoltaic cladding as a means for providing the electrical needs of buildings. Interest is also growing in the use of such cladding as a combined photovoltaic / solar thermal collector, using air as the 'coolant' for the photovoltaic arrays (Posnansky and Gnos, 1994; Takashima et al., 1994). The advantage here lies in the combination that makes the flat-plate collector part of the protective skin and mounting system for the photovoltaic cells, and also in the fact that the performance of photovoltaic cells improves as their temperature decreases (Duffie and Beckman, 1974; Brinkworth et al., 1997). Such 'multi-functional'
facades have the potential to improve the cost-effectiveness of solar collection; this occurs as a result of minimising the capital cost of a solar collector system by utilising a constructional feature for more than one role, or by using an otherwise necessary feature of the building (e.g. its weatherproof envelope) for collection (Holmes, 1994).

Concern over air quality in buildings, combined with the need to look for lower-energy techniques for the conditioning of internal environments, has led to the arrival of displacement ventilation systems (Alamdari et al., 1993) in the UK. Such systems have been used in mainland Europe for some time, and consist of supplying full fresh air at low level and low velocity to a zone. Heat sources within the zone then create rising thermal plumes which remove the heat load and contaminants. Since the displacement air is supplied at a temperature of about 19 °C, it may be possible to develop a solar-assisted displacement ventilation system. Thus, it is clear that many existing mechanically-ventilated buildings, together with existing and new ventilation systems, have the potential to utilise such solar assistance. However, it is the question of cost-effectiveness that requires answering in any such installation.

Concerns about possible climate change, opportunities that are likely to present themselves as a result of the trend towards sustainable architecture, and the changing attitudes that are likely to ensue amongst building owners suggest that it is once again appropriate to consider the potential of solar energy as a means for space heating in temperate climates. Key elements to be considered in this thesis are not only the multi-functionality of facades, but also the ability to use existing constructions (retrofit applications) as a means for improving cost-effectiveness and for increasing the uptake of such systems. The use of air as the heat transfer fluid will be examined in this context, due to its universal availability. The following sections trace the development and application to date of air heating solar collection, and describe the concept which will be the principal subject of this research.

1.3. Introduction to air heating solar collectors

The principle of the flat-plate solar collector is straight-forward. A flat plate solar collector usually consists of an absorber to intercept the solar radiation, a cover to reduce heat loss and to permit transmission of the radiation and
the necessary channels for heat transfer fluid (air or liquid) to pass through. Fluid flow through the collector is induced by a pump (for liquid) or by a fan (for air), the fluid removing heat from the collector. The heat removed can then be usefully employed (Duffie and Beckman, 1991).

Air heating solar collectors offer the following advantages over water systems:

- There is no risk of freezing with an air heating solar collector.
- There is no problem of corrosion due to leaking in an air-based system, as there would be with water-based systems. As a result of this, an air heating solar collector can function for many years without appreciable maintenance requirements.
- A substantial number of commercial buildings in the UK have ducted ventilation systems for heating and/or cooling purposes. This makes the integration of air heating solar collector systems as a pre-heating device straightforward.
- An air system can respond quickly in a direct air heating mode because of the relatively low thermal capacitance of the medium.
- They can be easily integrated within a roof or facade and offer additional insulation around a building in periods of low radiation.

However, their disadvantages are that leakage, if present, may go undetected, resulting in lower efficiency; they also require large surface areas and high air velocities to achieve reasonable energy collection rates.

1.4. Early history and development of solar air heating collectors

Most of the solar-assisted space-heating developments since 1940 have been in liquid solar heaters, and the commercial manufacture and sale of solar equipment after 1975 has been based mainly on liquid systems (Kreider, 1981).

The use of air as the medium for heat transfer between collector and storage (and also to the living space) in domestic-scale applications first commenced in 1944. A small dwelling in Boulder, Colorado, was retrofitted with a solar air heating system developed by Löf (1946). A solar heater of the overlapped
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glass-plate type and a pebble-bed heat storage bin were used in the Boulder residence. The auxiliary heating was provided by a gas boiler.

The first purpose-built solar air collectors were designed by Telkes (1949) for a house which was built in 1948 in Dover, Massachusetts. Vertical, glass-covered metal absorber plates served as solar collectors in the Dover house. The system has a heat store consisting of Glaubers salt. The auxiliary heating was provided by electric heaters.

It can be seen from the above examples that auxiliary heat was necessary and that the energy provided by the solar energy systems were acting as an assistance to the conventional heating systems.

Bliss (1955) constructed a matrix solar air heater (solar absorption in black cotton gauze) and a pebble-bed storage bin adjacent to a small house in Amado, Arizona. In 1957, Löf built a house in Denver, Colorado, equipped with what has become one of the most famous solar air heating systems; its fame derives from the good performance of the system due to its prolonged, maintenance free routine operation (18 years), and the low reduction in its energy output (22% in 18 years) which was caused by glass breakage of the interior clear glass plates and black glass plates in the overlapped plate air heating collectors (Duffie and Beckman, 1991; Jesch, 1981; Kreider, 1981). The installation comprises two banks of 300 ft² (27.9 m²) overlapped glass-plate type solar collector. The collectors are in two banks, angled up from the flat roof at a slope of 45° and facing due south. Pebble-bed heat storage, an exchanger for hot-water supply, and a auxiliary gas boiler were used. The system is still operating continuously. An energy balance of the Denver Solar House estimated for winter 1959 to 1960 (Duffie and Beckman, 1991) indicated that the solar energy collected contributed about 25% of the house's heating load (including water preheating and space heating). The above performance measurements indicate that it is possible for air systems to operate over many years with very low maintenance (Duffie and Beckman, 1991).

In 1975, a commercially-designed solar air-heating system was installed in a new experimental house at Colorado State University (CSU) in Fort Collins, Colorado, USA. The house is one of three identical structures in the CSU Solar Village, each structure being solar-heated by a different type of system.
The collector is a double-glazed covered system with air flow between the cover and the black metal absorber plate. Pebble-bed heat storage, auxiliary gas heat, domestic hot water by heat exchanger, and accessories, complete the system (Duffie and Beckman, 1991; Kreider, 1981). Extensive measurement was conducted on the system's performance. The 68.4 m² air-heating solar collector contributes 343620.5 MJ (9545 kWh) to the building (130 m² in floor area). This represented a contribution of 72% of the total heating load. The ratio of solar energy delivered to electrical energy supplied to operate the system was 13.5 for the heating season. This figure corresponded to 707 kWh of electricity consumption for the fan operation. This resulted in a net annual energy savings of 8838 kWh.

These early developments showed that a fairly good energy saving can be achieved due to solar energy utilisation by an air-heating solar collector. The Denver Solar House (Duffie and Beckman, 1991) provided extensive measurements on thermal energy savings while both thermal energy saving and electricity consumption (due to fan operation) in the CSU Solar Village experimental house (Duffie and Beckman, 1991) was reported. If 1.5 pkWh⁻¹ and 5.0 pkWh⁻¹ were assumed for gas and electricity purchase price, one would expect an annual cost saving of around £108. This amount is considered moderate in terms of cost saving.

1.5. Recent application of air heating solar collectors

1.5.1. General

Use of air to collect and transport heat offers many advantages over liquid-cooled collectors, as already stated. Large buildings, such as office buildings and schools, are frequently constructed with mechanical ventilation systems. As future buildings are likely to include natural and / or mechanical ventilation systems, the possibility of integrating a solar air collector into the system becomes increasingly important (Hastings, 1984).

The followings are some more recent examples of air-heating solar collectors applications.
1.5.2. Crop drying applications

The simplest and cheapest method of drying corn and hay is to use solar air heaters on the roof of barn buildings (Røstvik, 1992). Such solar air-heaters are usually made up of a single collector with a transparent plastic cover on the outside and a dark absorber beneath it, the structure forming the roof of the building. This type of installation has been in use in all Scandinavian countries (Tengesdal, 1991). Different solar collector geometries are also used for this purpose in North America (Löf, 1962) as well as in other parts of the world (Kreider and Krieth, 1981; Bharadwaj et al., 1986).

A 'solar barn' for grain and hay drying experiments in Scotland (Ferguson and Bailey, 1981) has been reported (Figure 1.1). Both the north and the south faces of the roof consisted of bare-plate (uncovered) collectors. The south wall, however, was designed as a suspended-plate (covered) collector, with air passing on both sides of the absorber (between the translucent cover and the collector and between the collector and back plates). The aim was to obtain the highest possible efficiency from the limited area of the wall that was available. Consideration of the practice led to the use of acrylic-coated aluminium sheeting of a dark brown trapezoidal section. The covering selected was of translucent glass fibre reinforced plastic with an outer UV filter layer of polyvinyl fluoride (directly riveted to the collector plate). The total designed collector area was 320 m² (101 m² in the south wall and 219 m² in the roof). Performance in terms of heat removal factor ($F_r$), collector efficiency factor ($F''$) or overall collector heat transfer coefficient ($U_L$) were not reported. However, observations of the energy collection rates indicate 12 kW (0.075 kW/m²) was typical for overcast conditions with 65 kW (0.4 kW/m²) as a peak value in bright sunshine. Since the study was an experimental one only, no mathematical model for general design use has been reported. Results for hay drying showed a relative energy saving of 30% with the use of solar energy. A collector efficiency of 50-51% can be estimated from the reported energy utilisation (figures given above).
1.5.3. Applications in residential and educational buildings

The other major application of air heating solar collectors is in domestic buildings. A block of 24 flats, built in the 1950's, and retrofitted with a roof-integrated air heating solar collector is in operation in Sweden. A 350 m² solar air-heating plant was integrated into the roof of the building. A fan induces flow of the solar-heated air in the cavity formed by the wall and the insulation. The heat is conducted through the walls and into the flats. An electric heater gives additional heating to the flats. The collector supplies 73000 kWh for space heating and 30000 kWh for domestic water heating annually. The building heating requirement is now only 31% of that before the retrofit (Røstvik, 1992).

A test house (Figure 1.2a) was constructed with a purpose-built air-heating solar collector (Johnson, 1981). The project was purely experimental and monitoring work was performed. The air-heating solar collector modules were roof mounted at an angle of 45° and faced due south. They were of a simple rear-duct type made of aluminium and each measures 4.3 m long by 0.54 m wide and were covered with 6 mm thick single glazing. The separation between the black anodised (non-selective) absorber and the rear plate was 20 mm. The underside of the collector was insulated with 80 mm thick fibreglass, attached directly to the rear plate. The total collector aperture area was 33.3 m². A heat store in the form of a vertical column of 'no-fines' concrete had been installed. The control strategy (Figure 1.2b) is such that priority is always given to solar collection, air being circulated through the collector array whenever the temperature differential between the rear
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collector plate and inlet air exceeds 10°C. The outlet air is then ducted either
to the bottom of the store, or if space heating is required, distributed directly
to the house. If space heating is required and there is no solar collection, the
system reverts to the store-to-house mode. No performance figures in terms
of $F_R$ and $U_L$ have been reported. However, performance of the collector
system (efficiency) was found to be low. In the period between January and
May of 1981, efficiencies of 12.4% to 14% were measured (Johnson, 1981).

In another application, a school building in south-east England had
thermosyphoning air panels (TAPs) installed as part of a refurbishment (Lo,
1993). Patented in the US (Morse, 1881), the system relies solely on natural
convection to transfer solar heat gain and is an integral element of the
building. A TAP operates in the same manner as the natural convection mode
of a Trombe-Michel wall (Trombe, 1977) but without the heat storage effect of
the masonry in a Trombe-Michel wall. Heat admission to the building may be
controlled by closing the inlet and outlet of the panel, thus suppressing the
thermosyphoning effect; or, heat admission can be further controlled by
opening a passage to an external vent, thus inducing a buoyant ventilation
exhaust flow. The back of the collector is insulated to inhibit conductive heat
transfer to and from the building. The performance of the system was
monitored. During the period between 10 October 1989 and 24 April 1990,
the contribution from TAPs on south (23 m²) and east (27.3 m²) facades were
1970 kWh and 498 kWh, respectively. A simple payback period of 46 years
was estimated. The estimation was based upon 10 TAPs supplementing gas
at 1.40 pkWh⁻¹ and 2 TAPs supplementing electricity at 6.31 pkWh⁻¹. These values represents an annual cost saving of £ 43.7.

In Germany, the Schopfloch Kindergarten, a single-storey, flat roof building, and has a 38 m² air-heating solar collector fitted on the southern slope for space heating. The collectors are coupled to a 20 m³ rock-bed storage. Heat is discharged to the rooms through radiant floors and walls, similar to the hypocaust of ancient Roman structures (Wilson and Sherlock, 1980).

The solar and auxiliary heating systems were designed to operate in the following manner: heat from the air collectors or from the storage is given preference over heat from the auxiliary system. The warm air is heated further if the actual inlet temperature deviates from the required set point, which is influenced by the ambient temperature. The additional heater for the wall inlet air is activated if the room temperature falls below 19.5 °C. A sensor measures the temperature of the outlet air from the floor heat system, and the controller opens a bypass flap if this temperature exceeds the maximum storage temperature. Manually controlled ventilation flaps in the northern slope of the roof allow individual conditioning in summer based on the users' requirements (Hastings, 1994). The site-built air collector also serves as a roof covering. The absorber is black foil, with a solar absorption factor of 0.97. Double polytetrafluoroethylene (PTFE) films ("Hostaflon") are stretched over an aluminium grid structure, below which is a gap of 150 mm through which air circulates. The overall solar transmission of the Hostaflon films (thickness 150 and 50 µm, respectively) is approximately 0.9 and the long-wave radiation transmission (at 50°C) is 0.02 (nearly opaque). The underside of the collector is insulated with 100mm glass wool. The collector performance was reported in terms of the following parameters: \((\tau\alpha)_z=0.75, \ F_\pi=0.8\) and \(U_\pi=6.8\ \text{Wm}^{-2}\text{°C}^{-1}\). This represents a maximum efficiency of 60% (Hastings, 1994).
1.5.4. The 'Solarwall' system

'Solarwall', an air-ventilation solar collector for pre-heating fresh air, has been installed on various types of buildings because of its thermal and cost effectiveness (Trimble, 1995). 'Solarwall' consists of different geometries. In one geometry, fresh ventilation air is drawn up the wall and into the building, the air flowing between a corrugated absorber and a fibreglass glazing. Another geometry consists of an unglazed wall painted dark brown. Air intake grilles mounted on the underside of a roof canopy capture solar heated air as it rises up the wall. In a third geometry, a perforated plate is used instead of a glazed cover. Air is drawn through the perforated-plate and ducted into the building (Carpenter and Kokko, 1991).

The applications of the unglazed version of the 'Solarwall' were mainly at industrial sites. Aveda Corporation, a beauty product manufacturer based in Minneapolis, USA, has a 116.8 m² Solarwall installed to assist space heating by preheating ventilation air. The collector was mounted 76 to 203 mm (3 to 8 inches) from the building's existing south facing wall. The results were an
energy saving of 130000 kWh annually, equivalent to a reduction of 90000 pounds of carbon dioxide per year (SEIA, 1995).

A glazed version of 'Solarwall' of 1877 m² area was installed at the Ford Motor Company plant in Oakville, Ontario, Canada. The system is made up of standard building wall components, including 38 mm of fibreglass insulation, corrugated steel wall cladding for the solar absorber, and translucent fibreglass-reinforced plastic glazing as the collector cover. An estimated peak efficiency of 45% was calculated (Carpenter and Kokko, 1991).

In another car factory application at Ford Motor Company plant, Chicago, USA, the installation (previously a glazed system but now replaced by a perforated-plate system) has an area of 5388 m² and was mounted on the outside wall of the factory. The new system has been operated since 1989. The efficiency of the solar collector varies from 41% to 87% depending on wind speed and temperature. The percentage of annual load met by solar energy was 14.8%. The payback period of the system was estimated to be only 2 years (Røstvik, 1992; SEIA, 1995).

The glazed version of the Solarwall system employs translucent fibreglass-reinforced polyester (FRP) as the glazing material (Carpenter and Kokko 1991). The drawback of using FRP is that it experiences losses in solar transmission of 1%, 3% and 11% when exposed to temperatures of 66°C, 93°C and 149°C, respectively, for 300 hours (Kohler et. al., 1979). Older versions of polycarbonate cover material had the disadvantages of ultra-violet
(UV) degradation and high transmittance in the infra-red (IR) (Kohler et al., 1979). However, with the new generation of polycarbonate cover material (Ariel Plastic Limited, 1990), a built-in layer that protects the material from UV degradation has been introduced and the transmittance in the IR range has been reduced (≈10%). These improvements have made the polycarbonate as more suitable for use as a solar collector glazing material, and thus it could be employed in an improved version of the 'Solarwall' system.

Due to the further problem of appearance and transmittance loss (Kohler et al., 1979), it was concluded that fibreglass-reinforced polyester (FRP) was not suitable as a solar collector glazing material. Therefore, it is necessary to consider more aesthetically-pleasing cover materials than those used for the original 'Solarwall'. Rain water can penetrate the perforated-plate system, if installed on roofs. Its applications are thus limited to vertical facades.

Although the perforated-plate version of 'Solarwall' has been reported to be efficient and that the payback period is reported to be short under northern American climatic conditions, its performance is unknown for European climatic conditions and, therefore, the potential annual benefit and, payback period of such a system are unknown.

Furthermore, there is no record of any mathematical model reported for any geometry of the 'Solarwall' type that could be used for design purposes. Performances reported for 'Solarwall' in the climatic conditions of the Northern American continent have been encouraging. It is considered worthwhile, therefore, to devise a modified or improved geometry for use in Europe and to investigate its performance in UK climatic conditions. Such an arrangement is discussed next, employing polycarbonate as an enhanced transparent cover system.

1.6. The proposed concept: A profiled steel air heating solar collector and its application in the UK

1.6.1. General

Profiled steel is widely used as an inexpensive cladding for many buildings in the UK, as well as in mainland Europe. With the development of more aesthetically-pleasing colours and finishes, and the introduction of a variety of
trapezoidal profiles, many types of buildings such as schools and colleges, offices, sports buildings, factories and hospitals now utilise this building product as a building envelope out of choice rather than cheapness (Brookes, 1990; British Steel, 1988a).

It is possible to adapt this type of cladding to become an air-heating solar collector by attaching a suitable transparent material to act as a cover over the cladding (Figure 1.5). By leaving a gap between the profiled steel (which acts as the absorber) and the transparent material (which acts as the cover), an air flow channel above the absorber is formed. The trapezoidal profile offers an enhancement to heat transfer due to the increased absorber surface area. A solar collector of this kind is called a structurally-integrated air-heating solar collector because it can be integrated onto either walls and roofs, which then act as 'multi-functional' facades, performing the dual role of weatherproof skin and energy collector.

![Figure 1.5: Possible structural alteration for solar energy utilisation](image)

The advantages of structurally-integrated air-heating solar collectors over existing profiled-clad buildings are:

- The building envelope would perform the dual function of weatherproof membrane and energy collector, thereby acting as a multi-functional facade, and enhancing the cost-effectiveness of construction. The heat that could otherwise be lost from the rear of an ordinary air-heating solar collector would then provide a heat gain to the attached structure (building) instead.
The arrangement would provide a reduction in U-value for the facade, resulting from the cover and the air space between the cover and the profiled cladding.

In view of the studies mentioned earlier, there may be potential in the UK for solar energy utilisation with covered profiled steel solar collectors. This is because:

- In normal, temperate environments, profiled steel coatings such as Colorcoat Pvf2 have a life expectancy of over 40 years, provided that it is properly maintained (British Steel, 1988). With special coatings such as Colorcoat Pvf2, profiled steel can withstand continuous temperatures as high as 120°C. Profiled steel cladding coated with these finishes should withstand the surface temperatures that would be experienced as the absorber of an air heating solar collector.

- A major retrofit opportunity exists, due to the large number of buildings that already utilise this type of cladding. There are over 100 million square metres of profiled cladding erected annually in the European Community (British Steel, 1994). Recent market figures for the UK show that steel and aluminium roof and wall cladding usage per annum was 12.5 million square metres, forming 17% of the total roof and wall market in the UK. The use of profiled metal cladding has also spread to business parks and office buildings which now form 13.6% (1.7 million square metres) of the total area of steel and aluminium cladding installation in the UK per annum (Ryan et al., 1994).

1.6.2. Proposed air heating solar collector as a preheating device

Solar panels which provide space heating by circulating air from the building through the collectors can only operate when the temperature of the solar heated air is above that of room air temperature; this restricts their use to only sunny days. However, when a solar panel is used to heat outside air, then any temperature rise of the air over ambient is useful energy; as a result, heat can be produced even on cloudy days when diffuse radiation is dominant.
In view of this, the proposed air-heating solar collector is considered to be operated as a pre-heater for the externally-drawn fresh air supply to the building. Such an air supply can be used for heating and/or ventilation. It is considered to be integrated into the roof and/or the wall of a building which is already clad with profiled metal cladding. The use of the proposed solar collector as an air pre-heater in this way offers a large, relatively inexpensive collector area. This aids exploitation of direct and diffuse radiation, the latter being an important consideration in view of the frequency of overcast sky conditions in the UK during the heating season.

Some earlier research on air-heating solar collectors had been conducted under UK conditions by BSRIA (Alper et al., 1985). Up to 1985, air-based solar heating systems were relatively untried in the UK. For commercially available rear-duct collectors, BSRIA found that a typical peak efficiency figure, $\eta_p$, (the efficiency when the air inlet temperature equals the ambient air temperature), was about 70%, suggesting that reasonable performance might be expected. However, the collector geometry being considered here (air-heating solar collector operating as an air pre-heater and constructed with profiled metal cladding as the absorber and polycarbonate sheeting as a cover) has not been tested. It is therefore necessary to investigate and quantify the performance, and to evaluate the energy performance of the collector and the building (of which it is an integral component) in the UK climate.

For the simplicity of retrofit and installation, the collector proposed consists of a transparent cover attached to the existing metal profile facade of the building, the latter acting as the absorber. A gap between the cover and the absorber forms a channel through which air can flow.

The cover material to be considered in the evaluation is of polycarbonate, with a triple wall structure. The advantage of this material is that it has a high short-wave transmissivity (68% for wavelength 0.4–0.8 $\mu$m) and a low long-wave transmissivity (less than 2% at around 9 $\mu$m). In addition, it offers some thermal insulation as a result of its multi-layered structure, and is a conventional building material, commonly used as a transparent facade for conservatories. A triple-layered structure offers better thermal resistance compared to a double-layered cover system.
1.6.3. Nature of application

Basically, any building that is constructed with profiled steel cladding as a facade can potentially use the proposed air heating solar collector system. Profiled steel-clad buildings, ventilated with ducted air systems, and where heating is the only form of space conditioning, are quite common and thus will form the basis of this study. Building types that utilise such systems are leisure centres, industrial estate factories and offices. While retail outlets are a common form of building that are clad with metal profile, they often simply heat outdoor air directly using gas heaters and then supply this air directly to the space. In addition, due to the nature of their business (retail through inexpensive premises), they are unlikely to invest in energy-conserving technology of the type proposed.

Owners of the remaining building types, such as sports centres, are more likely to invest in energy conservation features of the type proposed (Gibb, 1996). The Sports Council has produced many publications to promote energy conservation (Sports Council, 1985, 1987a, 1987b and 1987c), thus confirming their enthusiasm for conserving energy in the leisure sector. Therefore, sports centres will be taken as the building type to be studied in this work. It should be noted, though, that the energy saving principles will still apply to other premises that use profiled cladding.

Previous studies have reported on the use of profiled metal as the absorber of an air heating solar collector in the USA (Wright et al., 1979). The collector consists of a cover over the profiled absorber with air flow between them. It was shown that profiled metal is easily installed and functions well as an absorber plate. Yet another study (Kohler et al., 1979) reported on a glazed structure-integrated site-fabricated air heating solar collector, with air flow underneath a corrugated absorber. Both studies have proved that construction of roof-integral air heating solar collectors is possible and straight-forward.

As the proposed solar collector system is intended for retrofit application, requiring little alteration on the existing structure, a collector geometry with flow between the absorber and cover system is considered. A geometry such as a bare-plate solar collector (flow underneath absorber), if applied in a
retrofit application, will require major alteration on the existing structure and thus it is not desirable.

1.7. Collector modelling

The use of an ordinary tile roof and an uncovered (unglazed) metal profiled roof as uncovered air-heating solar collectors for the UK climate have been reported (Loveday, 1988). Although the air temperature rises produced by the tile roof structure were relatively modest (1.2°C to 1.4°C) compared with purpose-designed solar collectors, it was considered that economic benefit might be derived from the use of the roof as a pre-heater. The report also stated that since the modifications needed during roof construction involve little cost, the use of conventional roofs, and especially metal roofs, might be economically attractive. A mathematical model is also presented for the geometries reported. However, it was assumed that the temperatures of the front and rear ambient environments were equal, despite the fact that the collectors were supposed to operate as a building structure (where front and rear environment temperatures will differ). Therefore there is a need to derive a mathematical model for the proposed building integrated geometry (in which front and rear ambient temperatures can differ). Such a model will be of benefit for the analysis of multi-functional facades in general.

The simplified mathematical models derived by others, the most important of which is the Hottel-Whillier-Bliss (HWB) equation (Huang and Howell, 1985), can only treat collector thermal performance by assuming that front and rear ambient temperatures are equal. To date, the thermal performance of solar collectors installed with their rear side thermally coupled to the internal environment of a building, has been determined by using the following correlations, obtained by modifying the HWB model; Here account is taken of the differing front and rear environment temperatures by simple averaging:

$$\eta = F_R \left\{ (\tau \alpha)_c - U_h \left[ t_f - \frac{t_a + t_u}{2} \right] / I \right\}$$

where

- $F_R$ = heat removal factor (-)
- $I$ = irradiance (Wm$^{-2}$)
- $t_a$ = front ambient temperature (°C)
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\[
\begin{align*}
    t_r & = \text{rear ambient temperature (°C)} \\
    t_f & = \text{fluid temperature in fluid channel (°C)} \\
    U_L & = \text{overall heat loss coefficient (Wm}^{-2}\text{°C}^{-1}) \\
    (\tau_\alpha)_e & = \text{effective transmittance-absorbance product (-)}
\end{align*}
\]

The approach by Huang and Howell (1985) is not acceptable for the evaluation of the collector performance of facade-integrated systems, especially where the front and rear insulation level of the collector differs significantly, although the approach was used in the air window collector investigation by Onur et al. (1996) (where it was considered satisfactory because of the similarity between the front and rear insulation levels because both the front and rear surface that formed the collector were glass). As for the proposed operation, the rear of the collector is to be thermally coupled with the internal environment and that causes a reduction in rear heat loss. In some instances, however, heat gain through the rear insulation of the collector to the attached zone could be experienced. For accurate performance prediction, more detailed modelling of the thermal processes at the front and rear of the collector is needed. For such processes to be modelled, a model is needed where the assumption of differing temperatures at the front and rear of the collector is needed at the outset. A proper evaluation of the heat transfer coefficients for the solar collector thermal process should be carried out. Since the heat transfer coefficients are temperature dependent to some extent, it is necessary with the proposed collector geometry and operation that the temperature of each surface should be evaluated in order for the heat transfer coefficients to be estimated. Since there is no simple mathematical model of this kind that describes the thermal processes of the proposed solar collector geometry, there is a need to derive a new mathematical model for this purpose.

1.8. Outline of the research approach

In order to fully evaluate the performance of a wall / roof integrated, covered profiled steel solar collector system on a typical UK building, it is necessary to determine its thermal performance and efficiency characteristics. It is also necessary to derive a model of its thermal performance which permits differing ambient temperatures to be considered at the front and rear of the collector (i.e. outdoor / indoor environments). Existing collector models, based on the Hottel-Whillier-Bliss approach, consider front and rear temperatures to be equal. A new model, in which front and rear ambient temperatures can
differ, would be of more general application for the analysis of structurally-integrated collectors.

The proposed arrangement of a facade-integrated system would have the added advantage of reducing building heat loss because of the additional transparent cover, and air gap, thereby reducing the U-value of the structure. In addition to the estimation of system energy performance, it will also be necessary to carry out an economic assessment of such a system based on capital, operational, maintenance costs and payback period. In this way, the potential of such an installation in the UK climate can be assessed.

The programme of work conducted in this study was therefore as follows. The solar collection performance of a section of the proposed new collector system was measured in a controlled environment using a purpose-designed solar simulator. The measurements were then used to validate a theoretically-derived mathematical model of the thermal collector performance of a facade-integrated air-heating solar collector for use in this application area which takes into account the following:

- differing front and rear environment temperatures
- the natural convection mechanism within the cover system
- the corrugation of the profiled metal absorber

The conditions of front / rear temperature difference and of rear insulation level for which the standard Hottel-Whillier-Bliss analysis is inadequate, were identified. The mathematical model derived, and validated for this particular application by the laboratory measurements was then encoded within an existing building thermal simulation program in order to assess the thermal performance of a case-study building under UK conditions. TRNSYS was chosen as the simulation program because of its ability to accept user-defined HVAC components (such as the structurally-integrated collector) into its component library. In this way, a generalised simulation tool was created for use by designers to evaluate buildings incorporating multi-functional, facade-integrated air-heating solar collectors. The simulation tool was then employed in evaluating the energy performance of a typical sports centre building under typical UK weather conditions. The annual energy saving was assessed and the payback period was determined.
Finally, general guidance is offered as regards the design and operation of such a facade-integrated collector system in the UK climate.

1.9. Summary of research objectives

The research objectives may be summarised as follows:

- To derive a mathematical model of the thermal performance of a covered profiled steel air heating solar collector as a facade-integrated component in a building;
- To measure the conventional performance characteristics of such a collector and to use the data for model validation for this particular application;
- To encode the model within the TRNSYS component library and to assess energy saving and cost-effectiveness of such a system for a profiled steel building in the UK climate;
- To provide guidance as regards the design and operation of such a system in the UK climate.

The structure of the thesis is arranged as follows.

Chapter 2 provides a survey of existing mathematical models for air-heating solar collectors with geometries similar to that proposed in this study. The differing assumptions and correlations were examined in order to find out what is lacking in the existing models so as to aid the development of the proposed mathematical model in this study.

Chapter 3 gives details concerning the test simulator construction, materials used, positioning of probes, calibrations and methods of measurement.

Chapter 4 presents the conditions of application for the standard Hottel-Whillier-Bliss analysis as regards facade-integrated collectors. A comparison between the measured and predicted collector thermal performance for standard profiled cladding confirmed suitability of the model for this particular application. The other heat loss paths were identified and were then accounted for in the predicted performance accordingly. The measured and predicted surface temperatures were also compared.
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Chapter 5 is an assessment of the different types of buildings that are likely to implement the proposed air-heating solar collector, and justifies the selection of a typical sports centre as the chosen case study.

Chapter 6 begins with a survey of two building thermal simulation programs. Of these, TRNSYS was selected. A computer model of the building in which the collector performance was assessed was created. Validation of the computer model was made possible with published data. Different collector geometries were then tried with the building computer model so that the energy saving with each geometry could be compared and that the best geometry might be chosen. The pressure loss due to the collector and the associated ductwork was estimated and incorporated into the overall performance of the collector. The cost saving was then estimated. General guidance of the collector’s operation were laid out.

Chapter 7 presents the estimation of the payback period of the proposed building-integrated solar collector system operating in the UK climate. Account is taken of the interest and inflation rates to arrive at the likely payback period for such a system in the UK, and hence an estimate of cost-effectiveness. The findings are compared with similar figures, estimated for the 'Solarwall' system in North America.

In Chapter 8, conclusions are drawn, and further avenues for research are outlined.
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References


British Steel (1988), 'Roofing and Cladding in Steel: Colorcoat Technical Specifications - British Steel Strip Products', November

British Steel (1988a), 'Roofing and Cladding in Steel: A Guide to Architectural Practice', November

British Steel (1994), *Private Communication*


Gibb A. (1996), Private Communication, Department of Civil and Building Engineering, Loughborough University


Chapter 1 - Introduction


Morse E. S. (1881), *US Patent* No. 246, 626


Sports Council (1985), 'Ventilation: Airchange-on-Demand Cuts Costs', Energy Data Sheet No.4


Sports Council (1987c), 'Enclosure: Enclose Your Energy', Energy Data Sheet No.17


Total Environmental Action (1975), 'Solar Energy Home Design in Four climates', TEA New Hampshire

Chapter 1 - Introduction

Trombe F. (1977), 'Concrete Walls to Collect and Hold Heat', Solar Age, 2, p.13


Chapter 2 - Mathematical modelling of the structurally-integrated air heating solar collector

2.1. Literature survey of mathematical models of air heating solar collectors

2.1.1. Background

An air heating solar collector usually consists of a solar energy absorber that forms part of an air channel. In the case of the proposed air heating solar collector, the cover system and the absorber form the top and bottom of the air channel, respectively. The absorber is usually selected to absorb solar radiation at wavelengths 0.4–0.8 μm of the solar spectrum; the cover is selected to transmit solar radiation at the same wavelengths, but to reflect and/or absorb at the longer wavelengths (8–9 μm).

When the collector (with transparent cover) is exposed to solar radiation, the cover allows solar energy to pass through which in turn is intercepted by the absorber. The absorber will then be heated and, in turn, will re-transit heat by long-wave radiation to the bottom of the cover and by convection to the fluid. The cover is usually opaque to the re-transmitted long-wave radiation and most of the energy would then either be absorbed by the cover or reflected back to the absorber. The heat loss from the system to the surrounding environment is usually from the top and bottom of the collector, with the top loss usually being the main heat loss mechanism.

An air flow through the collector is usually introduced by a fan situated in ductwork connected to the collector outlet. When the collector is exposed to solar radiation, the air flowing in the channel is heated, and a temperature difference is created between the air on entry to, and exit from, the channel.

An air heating solar collector for building applications usually operates in the following manner:

- heating of fresh air by passing outdoor air directly through the collector, making it a ventilation air pre-heating device, or,
- heating of room air from a building by recirculating room air through the collector.
The air heating solar collector proposed in this study is to be operated as a pre-heater of outdoor fresh air.

There have been many mathematical models developed for the prediction of the thermal performance of air heating solar collectors. The mathematical models developed by Close (1963) and Whillier (1964) were among the earliest works. Since then, mathematical models for various geometries have been developed. The most common ones are (Duffie and Beckman, 1991):

- flow over the absorber and below the cover;
- flow both sides of the absorber (covered);
- geometries as above, but with corrugated absorber, vee-corrugated absorber or finned absorber;
- flow between overlapped glass absorbers (each with the first half of the length being transparent and second half of the length being opaque (blackened));
- flow across a matrix absorber.

Comparison of performances between solar collectors is usually determined by comparing the following performance indices:

- the efficiency factor, $F'$,
- the collector heat loss coefficient, $U_L$,
- the heat removal factor, $F_R$, and
- the transmittance-absorptance product, $(\tau\alpha)_e$.

These indices are particularly useful when comparing the performance of different collectors and for the study of a specific collector's performance as a function of changes in environment and in the collector design parameters. For each collector geometry, the above two indices ($F_R$ and $U_L$) can be derived by performing heat balances on the different components of the collector, such as the cover, the absorber and the heat transfer fluid. The equations are then solved to obtain the indices and the efficiency of the collector. Note that the indices $F'$ and $U_L$ are both functions of heat transfer coefficients.
Values for the indices $F'$ and $U_L$ can also be obtained experimentally, based upon the overall collector energy balance:

\[
\frac{\text{Rate of energy incident}}{\text{rate of useful energy collected}} = \frac{\text{rate of energy loss}}{}
\]

that is:

\[
IA(\alpha) = Q_u - U_L A(t_p - t_o)
\]  

...(2.1)

Now, the efficiency factor ($F'$) is defined as the ratio of the actual useful energy gain to the useful gain that would result if the collector absorber surface had been at the local fluid temperature. When the absorber plate temperature $t_p$ is replaced by the mean fluid temperature $t_f$, and Equation 2.1 is re-arranged, the following expression is produced (Duffie and Beckman, 1991):

\[
\eta = F'(\alpha) = \frac{U_L (t_f - t_o)}{IA}
\]  

...(2.2)

where the terms are defined in the nomenclature at the end of this chapter.

Since heat removal factor ($F_h$) is defined as the ratio of the actual heat transfer to the maximum possible heat transfer, and that the maximum heat transfer can be achieved when the absorber is at the same temperature as that of the inlet fluid (to minimise collector heat loss), the following equation (also known as the Hottel-Whillier-Bliss equation) is obtained (Duffie and Beckman, 1991):

\[
\eta = F_h[(\alpha) - U_L (t_i - t_o) / I_T]
\]  

...(2.3)

where the meaning of $\eta$ is defined in the nomenclature.

The heat removal factor, $F_h$, is related to $F'$ in the following manner (Duffie and Beckman, 1991):

\[
F_h = \frac{mC_p}{AU_L} \left[1 - \exp\left(-\frac{AU_L F}{mC_p}\right)\right]
\]  

...(2.4)

where the terms are defined in the nomenclature.
2.1.2. Mathematical models associated with the proposed solar collector

As mentioned before, a mathematical model of a solar thermal process can be derived by heat balances on different components of a solar collector system. However, assumptions are made so as to simplify the mathematical analysis. For example, it is common to assume that the solar absorption by the cover system is accounted for by multiplying a factor (1.02) to the absorptance-transmittance product.

The following is the outline of derivations of mathematical models relating to geometries and operations which have some similarities to the proposed system.

Duffie and Beckman (1974)

The model derived here was for a collector with flow between an infinitely thin cover and an insulated absorber. The mathematical analysis of a simple covered air-heating solar collector with flow between the cover and the absorber, reported by Duffie and Beckman (1974), began by eliminating the back loss mechanism. Employing the above assumption and carrying out heat balances on various components of the collector, comparison with equation 2.2, yielded the following expressions:

\[ U_L = \frac{U_t}{1 + (U_t h_3) / (h_2 h_3 + h_2 h_{23} + h_3 h_{23})} \] \hspace{1cm} \text{...(2.5)}

and

\[ F = \left[ 1 + \frac{h_{23} U_t}{h_{23} h_2 + h_3 U_t + h_3 h_{23} + h_2 h_3} \right]^{-1} \] \hspace{1cm} \text{...(2.6)}

where the terms are defined in the nomenclature.

To account for back losses, it was suggested (Duffie and Beckman, 1974) that the back loss coefficient, \( U_b \), be added algebraically to the top loss coefficient, \( U_t \). However, this assumes that absorber plate temperature is the
same as the cover temperature. It was thought that the resulting error should be small (Duffie and Beckman, 1974).

In the application being studied in this thesis, however, the rear of the collector was to be coupled to a building (forming an integral part of a roof or wall); it was thus considered that heat transfer from the collector rear to the rear environment had to be modelled more rigorously.

Parker (1981)

Parker (1981) has developed four mathematical models for solar air heaters with the following geometries:

- flow between cover and absorber (Figure 2.1a)
- flow both sides of the absorber (Figure 2.1b & e)
- flow underneath the absorber (Figure 2.1c & f)
- bare plate (Figure 2.1d)
- triangular duct collector (Figure 2.1g)

The following assumptions were made:

- the heat transfer process is steady-state,
- the air temperature, \( t_a \), is the same above and below the collector.
Chapter 2 - Mathematical modelling of the structurally-integrated air heating solar collector

Figure 2.1c: Typical air-heating solar collector with flow underneath absorber

Figure 2.1d: Typical bare plate air-heating solar collector

Figure 2.1e: As (b) with v-corrugated absorber

Figure 2.1f: As (c) with v-corrugated absorber

Figure 2.1g: Triangular duct collector
For the air-heating solar collector model developed by Parker (1981) with flow between cover and absorber plate (Figure 2.1a), the analysis began by heat balances on the absorber, the cover and the fluid. The following were obtained (per unit area):

for the absorber:

\[ I(T_{cc}) = h_2(T_2 - t_f) + h_{n12}(T_2 - T_1) + U_a(T_2 - t_a) \]  \hspace{1cm} ...(2.7)

for the cover:

\[ U_c(T_2 - t_a) = h_1(t_f - T_1) + h_{n12}(T_2 - T_1) \]  \hspace{1cm} ...(2.8)

for the fluid:

\[ q_u = h_2(T_2 - t_f) + h_1(T_1 - t_f) \]  \hspace{1cm} ...(2.9)

By solving the temperatures in terms of heat transfer coefficients and then substituting for \( q_u \) to compare with

\[ q_u = F'[I_r(\tau \alpha) - U_L(t_f - t_a)] \]  \hspace{1cm} ...(2.10)

the following expressions were obtained:

\[ U_L = \frac{(U_i + U_a)(h_1 h_2 + h_1 h_r + h_r h_2) + U_i U_a(h_1 + h_2)}{h_1 h_2 + h_1 U_i + h_r U_i + h_1 h_r + h_1 h_2} \]  \hspace{1cm} ...(2.9)

and

\[ F' = \frac{h_1 h_r + h_2 U_i + h_2 h_r + h_r h_2}{(U_i + h_r + h_1)(U_a + h_2 + h_r) - h_r^2} \]  \hspace{1cm} ...(2.10)

The 'vee'-corrugated designs (Figures 2.1e, f and g) were considered (Parker, 1981) to have the same equations for \( F' \) and \( U_L \) as that for an equivalent geometry but with the flat-plate absorber. However, it was suggested that the value of \( h_2 \) increases approximately with surface area of the absorber, provided the angle of the 'vees' is relatively large (about 50°). Thus, the new surface conductance is \( h_2 \approx \sin(\theta / 2) \) (see nomenclature).

Though this geometry has similarities to that of the proposed collector in this study, the proposed collector has a different geometry of profiled steel absorber. More importantly, however, it is to be operated with the front
ambient temperature different from the rear ambient temperature; the models Parker (1981) treat the front and rear ambient temperatures as being equal.

Peck and Proctor (1983)

A study of a structurally-integrated solar air heater has been carried out. Figure 2.3 shows the construction of the collector. A mathematical model was derived based on a single glazed collector having a flow channel of rectangular cross section. The empirical correlation used for the heat transfer coefficients from the cover, between the top plate and the cover and between the bottom plate and the top plate (that formed the air duct) were reported. The study has also reported a comparison between the predicted and measured performance in terms of outlet temperatures and efficiencies.

![Cross-section through collector](image)

Figure 2.3: Cross-section through collector, as reported by Peck and Proctor (1983)

The energy exchange from the back of the collector was based on an "effective roof space temperature", allowing for convection to the roof air space and for radiation exchange with the collector rear surroundings. Collector top losses were based on the outdoor ambient temperature for the convection heat transfer, and on a sky temperature of \((T_a - 12) \, ^\circ C\) for long-wave radiation heat transfer.

The air-heater mathematical model was based on a single glazed collector having a rectangular cross section. A steady state energy balance was set up for each component of the collector. The resulting equations were then solved
simultaneously and iteratively to obtain the steady state component temperatures.

The performance in terms of $F_R$ and $U_L$ was not reported. However, validation of the model was carried out by comparing the predicted values and the measured values of outlet temperature and efficiency. Each comparison was made under the same conditions of irradiance and ambient temperature.

The way in which Peck and Proctor (1983) accounted for the effect of the difference between front and rear temperatures on the thermal performance was not clearly explained, making application of their approach to other situations difficult. Since there is no expression derived for $F_R$ and $U_L$, and validation of the mathematical model was performed by comparing the estimated and measured outlet temperatures and efficiencies instead of the estimated and measured $F_R$ and $U_L$, it was considered that simple algebraic averaging of front and rear ambient temperatures of the collector was employed. This approach was considered to be not acceptable for the proposed mathematical model in this thesis as the effect of the difference between front and rear ambient temperature has not been made exploit.

Loveday (1988)

The geometry reported was a flat uncovered absorber plate with air flow beneath it. Heat transfer was considered to be steady-state and one dimensional. This geometry differs from that of the present study. However, the mode of operation of the collector is similar to the proposed system in that the collector absorber is the ordinary tile roof of a dwelling, that is, a structurally-integrated solar collector.

A mathematical model was developed and was validated by laboratory measurement but for the case of front and rear ambient temperatures being equal. Further validation with field test measurements was carried out. Here, the difference in environment temperatures at front and rear of the collector was accounted for only by inserting the average value of the front and rear temperatures in the model. No attempt was made to include this effect into the mathematical model itself; instead, the model presented was reduced to
the standard Hottel-Whillier-Bliss (HWB) format in order to obtain the conventional collector characteristics of $F^*$, $F_R$ and $U_L$.

As in other mathematical models, the effect of the difference between front and rear environment temperatures has not been rigorously analysed.

Validation has been carried out with data measured in an existing house in which the temperature difference of the collector between the rear ambient (8.4 °C) and the front ambient (7.0 °C) environment were only 1.4 °C. This temperature difference is considered small and that the approach for averaging the temperature (to take into account of the temperature difference) was acceptable, as can be seen later (Chapter 4).

Although the above mentioned approach used to account for the temperature difference was satisfactory for that specific collector geometry, it is not to be used in the proposed mathematical model. In the proposed collector application, however, the effect of the front and rear temperature difference has to be properly taken into account.

Garg (1991)

An analysis has been carried out which includes a mathematical model of an air-heating solar collector with flow between cover and absorber plate. The following assumptions were made:

- heat transfer is steady-state;
- ambient temperatures at the front and the rear of the collector are equal;
- inlet temperature is assumed to be the same as the ambient temperature.
The analysis began by carrying out heat balances on the outer and inner cover layers, absorber and fluid. Note that absorption by various layers of the cover system was accounted for. The following equations were obtained:

\[ S_1 I_T + h_1(T_2 - T_i) = h_{1a}(T_1 - t_a) \]  \( \text{(2.13)} \)

\[ S_2 I_T = h_2(T_2 - T_i) + h_{23}(T_2 - T_3) + h_{2f}(T_2 - t_f) \]  \( \text{(2.14)} \)

\[ S_3 I_T = h_{32}(T_3 - T_2) + h_{3f}(T_3 - t_f) + h_{3a}(T_3 - t_a) \]  \( \text{(2.15)} \)

\[ (mC_p / W)(dt_f / dy) = h_{2f}(T_2 - t_f) + h_{3f}(T_3 - t_f) \]  \( \text{(2.16)} \)

where

\[ S_1 = \alpha_1 \]
\[ S_2 = \tau_i \alpha_2 \]
\[ S_3 = \tau_1 \tau_2 \alpha_3 \]

By solving equations 2.13 to 2.15 for \( T_1, T_2 \) and \( T_3 \) and by integrating equation 2.16 (after substituting for \( T_2 \) and \( T_3 \)) within the limits \( t_f \rightarrow t_a \) to \( t_o \), as \( y \rightarrow 0 \) to \( L \), the outlet air temperature \( t_o \) was obtained. The efficiency of the collector was then obtained by using the relation:

\[ \eta = \frac{mC_p(t_o - t_o)}{I_T} \]  \( \text{(2.17)} \)

The way in which the corrugation was treated was to increase the absorptance of the absorber from 0.90 to 0.95. It is not clear as to how such a factor (0.95/0.90=1.06) was derived.
Since the difference in ambient temperatures on the front and rear is again not accounted for, the model is not suitable for the proposed solar collector system.

Ong (1995)

Ong (1995) has developed a mathematical model for an air solar heater with flow between cover and absorber plate. The approach was similar to Garg (1991), in which the solar absorption by cover layers was accounted for in the analysis.

The useful heat transfer to the air was assumed to occur uniformly along a collector of length \( L \). In other words, instead of using

\[
q = \frac{dt_f}{dy} \frac{m C_p}{W}
\]

...(2.18)

to determine the fluid outlet temperature, the following was used:

\[
t_o = \frac{q W L}{m C_p} + t_i
\]

...(2.19)

The mean fluid temperature is then equal to the arithmetic mean:
\[ t_f = (t_i + t_o) / 2 \]  \hspace{1cm} \text{(2.20)}

This assumption implies that the air temperature varies linearly along the collector, although in reality this is only true for 'short' collectors (Ong, 1995). So for a long collector, it was assumed that the collector could be divided equally into a finite number of short collectors, or sections.

The procedure to determine the surface temperatures and efficiency was iterative, as follows:

- The surfaces and mean fluid temperatures of the first section (of a long collector) were initially guessed and specified to be equal to the ambient temperature.
- The heat transfer coefficients were evaluated according to the initially-guessed temperature values.
- An iterative process was then set up to enable the mean temperatures (of surfaces and outlet) for the collector to be calculated.
- The newly-calculated temperature values were then compared with the previously-assumed ones. The iterative process was repeated until all consecutive mean temperatures differed by less than 0.01 °C.
- Another section of collector with a length equal to the previous one was then added to the end of the first collector. The temperature conditions at the inlet of the second section were assumed equal to the outlet temperature conditions of the previous section. The iterative procedure was repeated to determine the mean temperatures for the next section.
- Additional sections were then added until the required overall length of collector was considered.

The reason for carrying out this procedure was that predictions of mean wall and air stream temperatures for a collector of any length could be obtained, without involving the complexity of differential equations in the analysis.

In the case of the mathematical model for this study, the predictions of temperatures on different surfaces and the fluid outlet temperature are required for building thermal simulation purposes. Ong's approach for determining the outlet temperature, especially for a long collector, is considered to be inadequate for this application because, for a long collector.
By assuming that the fluid temperature is the average temperature of that for the inlet and outlet air, error will be introduced, however small in a 'short' collector. If a collector system consists of elements of 'short' collectors sections (connected in series), as assumed by Ong (1995), a small error introduced in the fluid outlet temperature on the first section would lead to significant error in the final outlet temperature prediction. Since the collector in this study is to be used for pre-heating fresh air for utilisation by the existing air heating system, and that the energy saving due to the collector is directly related to the collector outlet temperature, the collector outlet temperature needs to be more accurately modelled. In addition, Ong's model retains the assumption that front and rear ambient temperatures are equal.

2.1.3. The need for of a new mathematical model

From the survey conducted above, it is clear that none of the existing mathematical models for flat-plate air heating solar collectors is directly suitable for the proposed application, though there are features which can be adopted from the existing models. The principal reason for their lack of suitability is the assumed equality of front and rear ambient temperatures. Therefore, a new mathematical model is needed which better accounts for the situation of a structurally-integrated solar collector, that is, for the more general case of differing front and rear ambient temperatures. It is also necessary to assess the conditions of front / rear temperature difference and rear insulation level for which such an approach is necessary.

In order to do this, it is necessary to account for the effect of differences between the front and rear ambient temperatures; this requires a special arrangement in the mathematical analysis (see later). Such a model can then be encoded into a building thermal simulation program for predicting building energy performance.

2.2. Model derivation

2.2.1. Model requirements and assumptions

Whether a solar collector is to be modelled as a steady-state or transient operation depends on the thermal capacity of the solar collector system.
Even though transient energy balance equations for various components of the collector can be developed to almost any desired degree of accuracy, the driving forces such as solar radiation, wind speed, and ambient temperature are usually known only at hourly intervals. This means that any predicted transient behaviour within the hourly intervals can only be approximate, even with extensive analysis (Duffie and Beckman, 1991).

The collector system being investigated consists of a covered profiled steel building facade with insulation at the back; it can thus be regarded as a thermally 'lightweight' construction. Furthermore, it was shown (Duffie and Beckman, 1991) that for a collector with a time constant of 15 minutes, the effect of transient operation is insignificant. The time constant of the proposed system was measured (see Chapter 3) and was found to be about 5 minutes. Thus, a steady-state approach was considered to be adequate for this analysis. Also, since the proposed model was to be used in a building thermal simulation program in which hourly weather data is provided, this further justifies the decision to adopt a steady-state approach.

Although the long-wave radiation transmission between the absorber and the sky would be small due to the proposed use of polycarbonate material for the cover system, it was decided that the effect of the long-wave radiation exchange mechanism should be included in the proposed mathematical model to cater for the use of other types of cover systems, and hence preserve generality.

Parker et al. (1993) had reported that for a 60° vee-corrugated absorber (doubled area), the increase in heat transfer coefficient per unit aperture area is generally in the range 50-80% as compared to the rectangular duct for collectors of equal length and equal pressure drop. Other studies (Sparrow and Lin, 1962; Hollands, 1963 and Loth, 1977) have attempted to account for the effect of vee-corrugation in terms of increase in surface absorptivity. The previously discussed work, however, is for 'vee'-grooved absorbers. The proposed collector has an absorber consisting of a trapezoidal profile with flat sections. In the absence of a study for such a trapezoidally-profiled absorber, it was considered that a simple area enhancement method ($K$ as defined in Figure 2.4) would be adopted here for including the effect of such corrugations on heat transfer. This area enhancement approach is similar to that applied in a study by Garg et al. (1983).
Since the temperatures on various surfaces in the collector had to be determined for building thermal simulation purposes as explained earlier, appropriate modelling of the effects of front and rear temperature difference was needed.

The following will therefore be included in the proposed model:

- A steady-state heat transfer mechanism is to be adopted.
- Long-wave radiation heat transfer mechanism between absorber and sky is to be included.
- A simple area enhancement, based on absorber-cover area ratio, is to be applied to the convection heat transfer mechanism between the absorber and the air stream, and to the radiation to the heat transfer mechanism between the absorber and the cover inner surface.
- The approach for solving the heat transfer terms and the temperature terms, used in Ong's (1995) study, is to be adopted. Only by solving the equations iteratively can the values for both heat transfer coefficients and temperatures be determined, because of the dependency of one upon the other.

Having studied the approach of previous works, the following are also considered to be necessary:

- The back heat loss mechanism has to be included directly at the outset of the mathematical analysis during model development.
- Owing to the way in which the proposed collector is to be used (wall / roof integrated unit), it is essential that the effect of differing ambient temperatures at front and rear of the collector be accounted for at the start of the mathematical analysis.
- A single effective transmittance-absorptance product term is assumed for the proposed model (see Duffie and Beckman, 1991).
- Since the collector system in this study can consist of large surface areas (building facades), significant flow lengths could be involved; thus, a mathematical model that assumes the inlet temperature of a 'module' being equal to the outlet temperature of a previous 'module' in a collector system is not appropriate for this study. Increase in fluid temperature therefore has to be modelled using an integration approach. The simple
assumption of fluid temperature being the average of that for inlet and outlet is not considered acceptable on the grounds of accuracy.

2.2.2. Mathematical derivation

The derivation begins by carrying out heat balances on the component surfaces. The mechanisms considered are shown in Figure 2.4.

For a unit width, \( \delta x \), of collector, the following energy balances apply:

**Cover**

\[
U_i(T_1 - t_o) + h_i(T_2 - t_f) = Kh_{r23}(T_3 - T_2)
\]  
...

**Absorber**

\[
U_b(T_3 - t_o) + Kh_3(T_5 - t_f) + Kh_{r23}(T_3 - T_2) + Kh_{r34}(T_3 - T_4) = I_f(\alpha) \varepsilon
\]  
...

**Fluid**

\[
\frac{mC_p}{\delta y} \frac{dt_f}{dy} = h_2(T_2 - t_f) + Kh(T_3 - t_f)
\]  
...

Figure 2.4: Heat transfer mechanism in the proposed mathematical model
Rearranging Equations 2.21 to 2.23 to give $T_2$, $T_3$ and $\frac{dt_f}{dy}$ yields:

$$T_2 = -\frac{U_b}{P_1(U_i + h + Kh_{23})} \frac{t_a}{t_f} + \frac{1}{P_1(U_i + h + Kh_{23})} \left( \frac{h}{h_{23}} + \frac{h}{U_i + h + Kh_{23}} \right) t_f$$

$$+ \frac{1}{P_1(U_i + h + Kh_{23})} \left[ \frac{U_i}{U_i + h + Kh_{23}} - \frac{Kh_{23}}{P_1} \right] t_a$$

$$- \frac{1}{P_1(U_i + h + Kh_{23})} I_T(\tau \alpha)_e$$

$$T_2 = -\frac{U_b}{P_1(U_i + h + Kh_{23})} \frac{t_a}{t_f} + \frac{1}{P_1(U_i + h + Kh_{23})} \left( \frac{h}{h_{23}} + \frac{h}{P_1(U_i + h + Kh_{23})} \right) t_f$$

$$+ P_1 t_a - \frac{1}{P_1(U_i + h + Kh_{23})} I_T(\tau \alpha)_e$$

$$T_3 = -\frac{U_b}{Kh_{23}P_1} \frac{t_a}{t_f} - \frac{1}{P_1} \left( \frac{U_i}{U_i + h + Kh_{23}} + \frac{h_{23}}{h_{23}} \right) t_a$$

$$- \frac{1}{P_1} \left( \frac{h}{h_{23}} + \frac{h}{U_i + h + Kh_{23}} \right) t_f - \frac{1}{Kh_{23}P_1} I_T(\tau \alpha)_e$$

$$T_3 = -\frac{U_b}{Kh_{23}P_1} \frac{t_a}{t_f} - \frac{P_1}{P_1} t_a - \frac{P_1}{P_1} t_f - \frac{1}{Kh_{23}P_1} I_T(\tau \alpha)_e$$

$\ldots(2.24)$

$\ldots(2.25)$
\[
\frac{dt_f}{dy} = \frac{W}{mC_p} \left\{ - \frac{h_s U_s}{P_1(U_s + h S + Kh_{23})} \frac{h_s U_s}{h_{23} P_1} t_a \right. \\
+ \left\{ \frac{h_2}{U_s + h S + Kh_{23}} - \frac{Kh_{23} \left( U_s + h S + Kh_{23} \right)}{P_1(U_s + h S + Kh_{23})} - 1 \right\} t_f \\
+ Kh_3 \left\{ - \frac{1}{P_1} \left( \frac{h_3}{U_s + h S + Kh_{23}} + \frac{h_2}{h_{23}} \right) - 1 \right\} t_a \\
+ \left\{ \frac{h_2}{P_1(U_s + h S + Kh_{23})} - \frac{h_3}{P_1 h_{23}} \right\} l_f(\tau \alpha) \varepsilon \\
\left. \right\} \\
\frac{dt_f}{dy} = \frac{W}{mC_p} \left\{ - \frac{h_s P_3}{P_1} t_a + \left\{ \frac{1}{U_s + h S + Kh_{23}} \left( h_2 - \frac{Kh_{23} P_3}{P_1} \right) - 1 \right\} t_f \\
- Kh_3 \left( \frac{P_3}{P_1} + 1 \right) t_a + h_2 \left\{ \frac{1}{U_s + h S + Kh_{23}} \left( U_s - \frac{Kh_{23} P_2}{P_1} \right) \right\} t_a \\
- \frac{Kh_3 P_2}{P_1} \left\{ t_a - \frac{P_3}{P_1} l_f(\tau \alpha) \varepsilon \right\} \\
+ \left( h_2 P_3 - \frac{Kh_3 P_2}{P_1} \right) l_f - \frac{P_3}{P_1} l_f(\tau \alpha) \varepsilon \right\} \\
\frac{dt_f}{dy} = \frac{W}{mC_p} \left\{ - \frac{h_s P_3}{P_1} t_a + P_6 t_f + P_7 t_a - \frac{P_3}{P_1} l_f(\tau \alpha) \varepsilon \right\} \\
\left(2.26\right) \\
\right. \\
\int \frac{dt_f}{P_7 t_a - \frac{P_3}{P_1} l_f(\tau \alpha) \varepsilon - \frac{U_s P_3}{P_1} t_a} = \int \frac{W}{mC_p} : dy \\
\frac{t_f}{P_6 P_1 t_a - \frac{P_3}{P_6} t_a + \frac{P_3}{P_6 P_1} l_f(\tau \alpha) \varepsilon + \exp\left( \frac{P_9 W_y}{mC} \right) C_1} \\
\left(2.27\right)
where $C_1$ is a constant.

Boundary conditions are applied by noting that by definition; at $y = 0$, $t_f = t_i$ and at $y = L$, $t_f = t_o$.

$$t_f = \frac{U_h}{P_6 P_1} t_o - \frac{P_7}{P_6} t_a + \frac{P_3}{P_6 P_1} I_f (\tau \alpha)_c + C_1 \quad \ldots (2.28)$$

$$t_o = \frac{U_h}{P_6 P_1} t_o - \frac{P_7}{P_6} t_a + \frac{P_3}{P_6 P_1} I_f (\tau \alpha)_c + \exp \left( \frac{P_6 W L}{mC} \right) C_1 \quad \ldots (2.29)$$

Combining Equations 2.28 and 2.29 yields:

$$t_v = \frac{U_h}{P_6 P_1} t_o - \frac{P_7}{P_6} t_a + \frac{P_3}{P_6 P_1} I_f (\tau \alpha)_c + \exp \left( \frac{P_6 W L}{mC} \right) \left[ t_i - \frac{U_h}{P_6 P_1} t_o - \frac{P_7}{P_6} t_a - \frac{P_3}{P_6 P_1} I_f (\tau \alpha)_c \right] \quad \ldots (2.30)$$

The mean fluid temperature throughout the channel is:

$$t_f = \frac{1}{L} \int_0^L t_f \cdot dL$$

$$t_f = \left[ \frac{U_h}{P_6 P_1} t_o - \frac{P_7}{P_6} t_a + \frac{P_3}{P_6 P_1} I_f (\tau \alpha)_c \right] \left\{ 1 - \frac{mC}{P_6 W L} \left[ \exp \left( \frac{P_6 W L}{mC} \right) - 1 \right] \right\}$$

Efficiency is defined as:

$$\eta = \frac{mC}{I_f W L} (t_o - t_i) \quad \ldots (2.32)$$

Substituting Equation 2.29 into Equation 2.32 and simplifying gives:

$$\eta = \frac{mC}{I_f W L} \left[ 1 - \exp \left( \frac{P_6 W L}{mC} \right) \right] \left\{ \frac{P_3}{P_6 P_1} I_f (\tau \alpha)_c + \frac{U_h}{P_6 P_1} t_o - \frac{P_7}{P_6} t_a - t_i \right\} \quad \ldots (2.33)$$
The collector heat removal factor is conventionally defined as follows:

$$F_h = \frac{mC}{WLU_L} \left[ 1 - \exp \left( -\frac{WLU_L F'}{mC} \right) \right] \quad \ldots(2.34)$$

where $U_L$ is the total heat loss factor referenced to ambient environment, $t_a$.

Since in this case a front and rear ambient temperature difference exists, i.e. $(t_a \neq t_a')$, an equivalent temperature, $t_a'$, is introduced in the standard Hottel-Whillier-Bliss (HWB) equation to replace the ambient temperature, $t_a$. An alternative HWB equation is formed:

$$\eta = \frac{mC}{I_TWU_L} \left[ 1 - \exp \left( -\frac{WLU_L F'}{mC} \right) \right] \left[ \frac{I_T(t\alpha)_e}{U_L} - t_i + t_a' \right] \quad \ldots(2.35)$$

Substitute equation (2.34) into equation (2.35),

$$\eta = \frac{mC}{I_TWU_L} \left[ 1 - \exp \left( -\frac{WLU_L F'}{mC} \right) \right] \left[ \frac{I_T(t\alpha)_e}{U_L} - t_i + t_a' \right] \quad \ldots(2.36)$$

Comparing the coefficients for the $I_T(t\alpha)_e$ and $U_L F'$ terms within the exponential expression for Equations 2.34 and 2.36 yields:

$$U_L = \frac{P_6 P_1}{P_3} \quad \ldots(2.37)$$

and

$$F' = -\frac{P_6}{U_L} \quad \ldots(2.38)$$

In order to compare the temperature terms between Equations 2.46 and 2.50, the following condition has to be fulfilled:

$$t_a' = \frac{P_6 P_1}{P_6 P_1} t_a' - \frac{P_6}{P_6} t_a \quad \ldots(2.39)$$

The above derivation suggests that the collector, having the front and rear ambient environment at $t_a$ and $t_a'$, respectively, would give the same thermal
efficiency as that for the same collector when subjected to an environment where the surrounding ambient temperature is the same as the equivalent ambient temperature, \( t_a^* \), with everything else (mass flow rate, irradiance, etc.) being the same.

Expanding out Equations 2.37 and 2.38 yields:

\[
U_L = \frac{K(U'_t + U_h)(h_2h_{23} + h_3h_{23} + Kh_{23}) + U'_tU_h(h_2 + Kh_2)}{K(h_3U'_t + U_h + Kh_{23} + h_2h_{23}) + K(h_2U'_t + U_h + Kh_{23} + h_2h_{23})}
\]  
\[+ \frac{Kh_{23}[U'_t(h_2 + Kh_2) + Kh_{23}(h_2 + Kh_3) + Kh_{2}h_3]}{K(h_3U'_t + U_h + Kh_{23} + h_2h_{23}) + K(h_2U'_t + U_h + Kh_{23} + h_2h_{23})}
\]  
\[= (2.40)
\]  

and

\[
F^* = \frac{K(h_2U'_t + U_h + Kh_{23} + h_2h_{23})}{(U'_t + U_h + Kh_{23})(U'_t + U_h + Kh_{23} + Kh_{23})(U'_t + U_h + Kh_{23})}
\]  
\[= (2.41)
\]

where

\[
U'_t = U_t + \frac{2U_eA_e}{LW} \quad \ldots (2.42)
\]

\[
U_e = \frac{1}{0.06 + \frac{d_e}{k_e}} \quad \ldots (2.43)
\]

\[
U_t = \left[ \frac{1}{K_{cover}} + \frac{1}{h_1 + h_{r1}} \right]^{-1} \quad \ldots (2.44)
\]

\[
K_{cover} = \left[ \frac{1}{h_{cl1a} + h_{r1a}} + \frac{1}{h_{cl2a} + h_{r2a}} \right]^{-1} \quad \ldots (2.45)
\]

\[
U_h = \left[ \frac{d_h}{k_h} + \frac{1}{h_4 + h_4} \right]^{-1} \quad \ldots (2.46)
\]

Outer surface temperatures \( T_i \) and \( T_4 \) can be determined from:

\[
T_i = \frac{K_{cover}T_2 + (h_1 + h_{r1})t_a}{h_i + h_{r1} + K_{cover}} \quad \ldots (2.47)
\]

and
$$T_4 = \frac{k_2 T_3 + (h_4 + h_{ra}) T_3}{h_4 + h_{ra} + \frac{k_2}{d_2}}$$

... (2.48)

2.3. Heat transfer correlations (and their conditions of use)

2.3.1. Overall collector arrangement

To estimate the convection heat transfer coefficient at the cover outer surface, there are two correlations that could be used. The following expression reported by McAdams (1954) is valid for wind speed of less than 5.0 ms$^{-1}$.

$$h_1 = 5.7 + 3.8 V_p$$

... (2.49)

Note that the same correlation was used when evaluating the convection heat transfer in the laboratory work. The use of the above correlation was appropriate since it may be applied to air flows over and parallel to a surface.

The flow over a collector mounted on a house is not always well represented by wind tunnel tests of isolated plates. Mitchell (1976) investigated the heat transfer from various (animal) shapes and showed that many shapes were well represented by a sphere when the equivalent sphere diameter is taken as the cube root of the volume. The heat transfer obtained in this manner is an average that includes stagnation regions and wake regions. Duffie and Beckman (1991) suggested that a similar situation might be anticipated to occur in solar collector systems. Since Mitchell (1976) suggested that the wind tunnel results of these animal tests should be increased by approximately 15% for outdoor conditions, assuming a house to be a sphere, the Nusselt number (taking into account the 15% increase) may be expressed as:

$$N_u = 0.42 \text{ Re}^{0.6}$$

... (2.50)

where the characteristic length is the cube root of the house volume ($L''$).
It is apparent that the calculation of wind-induced heat transfer coefficients is not well established (Duffie and Beckman, 1991). Until additional experimental evidence becomes available, Duffie and Beckman (1991) recommended that Mitchell’s (1976) correlation (Equation 2.50) should be used for the estimation of convection coefficient at the front surface of roof-mounted collectors. By substituting the characteristic length ($L''$), air speed and kinematic viscosity into Mitchell’s correlation (Equation 2.50), the following expression results:

$$h = \frac{8.6V^{0.6}}{L''^{0.4}}$$  \hspace{1cm} ...(2.51)

Consequently, it appears that a minimum value of convection coefficient of approximately 5 Wm$^{-2}$°C$^{-1}$ occurs in solar collectors under still air conditions (Duffie and Beckman, 1991). Therefore, the following correlation will be used in this work for estimating the collector front convective heat transfer coefficient in outdoor conditions:

$$h = \left\{ \left( \frac{8.6V^{0.6}}{L''^{0.4}} \right) \right\}_{\text{max}}$$  \hspace{1cm} ...(2.52)

where the terms are defined in the nomenclature.

The radiation heat transfer coefficient between the outer cover surface and the front ambient environment was evaluated from an expression given by Duffie and Beckman (1991) for the laboratory validation case:

$$h_{\text{r, i}} = \varepsilon \sigma \left[ (T_i + 273)^2 + (t_o + 273)^2 \right] (T_i + t_o + 546)$$  \hspace{1cm} ...(2.53)

For outdoor conditions, a number of correlations have been proposed to relate the effective blackbody sky temperature to the ambient temperature and other meteorological variables. The investigations by Swinbank (1963) and by Whillier (1967), who related sky temperature to the local air temperature, are among the most well known. Berdahl and Martin (1984) used extensive data from the USA (Tucson, AZ; San Antonio, TX; Gaithersburg, MS; St. Louis, MO; West Palm Beach, FL; and Boulder City, NV - 6 sites for 57 months) to develop a correlation which relates the effective sky temperature to the dew point temperature, dry bulb temperature, and the hour from midnight:
\[
T_s = T_a \left[ 0.711 + 0.0056 T_{dp} + 0.000073 T_{dp}^2 + 0.013 \cos(15 t_{ma}) \right]^{1.4} \quad \ldots \tag{2.54}
\]

where the terms are defined in the nomenclature and the cosine term is in degrees.

The correlation reported by Berdahl and Martin (1984) is considered to be more appropriate because the data used for extrapolating the correlation covered a wide range of dew point temperature (-20°C to 30°C) and was taken from a variety of climatic regions, the correlation is more 'universal' than the previous ones. Therefore, the correlation by Berdahl and Martin (1984) is used here for estimating the equivalent sky temperature and hence the radiation heat transfer coefficient between the cover and sky.

The Nusselt number for the collector air channel was evaluated from Kays' (1966) correlation for fully developed turbulent flow between flat plates with one side heated:

\[
N_u = 0.0158 \text{Re}^{0.8} \quad \ldots \tag{2.55}
\]

Entrance region effects were accounted for using the following correlation (Duffie and Beckman, 1991):

\[
N_u = N_u_{fd} \left[ 1 + \left( \frac{d_h}{L} \right)^{0.7} \right] \quad \ldots \tag{2.56}
\]

The convection coefficients \( h_2 \) and \( h_3 \) in the air flow channel may then be found from:

\[
h_2 = h_3 = \frac{N_u \cdot k}{d_h} \quad \ldots \tag{2.57}
\]

When no air flows through the collector channel, the heat transfer mechanism between the absorber and cover system is considered to be that in an enclosure. The correlation applied is the same as that used for natural convection within the cover system (see section 2.3.2).
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\[ h_{r,23} = \sigma \left[ \frac{(T_2 + 273)^2 + (T_3 + 273)^2}{\varepsilon_e} \frac{1}{\varepsilon_3} - 1 \right] (T_2 + T_3 + 546) \] ... (2.58)

At the rear surface of the collector, natural convection is considered to occur for the situation similar to that of a wall in a room. The convective heat transfer coefficient for this situation, \( h_4 \), can be calculated from an expression by Holman (1981):

\[ h_4 = 1.42 \left( \frac{T_4 - t_{a,4}}{L} \right)^{0.25} \] ... (2.59)

The radiation heat transfer coefficient between the insulated backing and the indoor environment, \( h_{r,4} \), is given by:

\[ h_{r,4} = \sigma \varepsilon_4 \left[ (T_4 + 273)^2 + (t_{a,4} + 273)^2 \right] (T_4 + t_{a,4} + 546) \] ... (2.60)

2.3.2. The multi-cover system

The transparent cover employed in this work consisted of a triple layer arrangement. Heat transfer within the triple-layer cover (Figures 2.4 and 2.5) was considered as follows. For heat transfer within the sealed cover system (between the partitions), both natural convection and radiation heat transfer between the layers were assumed. Due to the thinness of the polycarbonate partitions (both vertical and horizontal), conduction (through the layers of conductance of 111.11 \( \text{Wm}^{-2}\text{C}^{-1} \)) would be small compared to heat transfer by convection and radiation (equivalent conductance of about 4.804 \( \text{Wm}^{-2}\text{C}^{-1} \)), and therefore conduction across the cover was assumed to be negligible.
Several attempts had been carried out to formulate natural convection correlations within enclosures for solar collector multi-layered covers. Hollands et al. (1976) derived from experiment the following correlation for the case of air in an enclosure (between two parallel plates) for the range of inclination angles $0^\circ$ to $70^\circ$ to the horizontal:

\[
Nu = 1 + 1.44 \left[ 1 - \frac{1708}{Ra \cdot \cos \phi} \right]^{1/3} \left[ 1 - \frac{(\sin 1.8 \phi)^{1/6} 1708}{Ra \cdot \cos \phi} \right]^{1/3} + \left[ \frac{(Ra \cdot \cos \phi)}{5830} \right]^{1/3} - 1 \]  
\]

...(2.61)

where $[ ]^+ = 0$ when $[ ] < 0$.

ElSherbiny et al. (1982) have derived two correlations, one for an enclosure at an inclination of $60^\circ$ to the horizontal (Equation 2.62) and the other at $90^\circ$ to the horizontal (vertical layers) (Equation 2.64).

\[
Nu_{60} = \left\{ \left[ 1 + \left\{ \frac{0.0936Ra^{0.314}}{(1 + G)} \right\}^{7/7} \left( 0.104 + \frac{0.175}{AR} \right) Ra^{0.283} \right\}_{\max} \right\}^{1/7}  
\]

...(2.62)

where

\[
G = 0.5 \left/ \left[ 1 + (Ra / 3160)^{0.6} \right]^{0.1} \right. \]  
\]

...(2.63)

and
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\[ Nu_{90} = \left\{ 0.0605 Ra^{1/3}, \left\{ \frac{1.04 Ra^{0.294}}{1 + (6310/Ra)^{1.36}} \right\} ^{1/3}, 0.242 \left( \frac{Ra}{\nu R} \right)^{0.272} \right\}_{\text{max}} \quad (2.64) \]

where

\[ Ra = \frac{g \beta \Delta T d^3 \Pr}{\nu^2} \quad (2.65) \]

ElSherbiny et al. (1982) suggested that the free convective heat transfer for inclinations between those stated above can be determined by interpolation in the following manner:

\[ Nu_\phi = \left[ (90 - \phi) Nu_{60} + (\phi - 60) Nu_{90} \right] / 30 \quad (2.66) \]

Another correlation for free convection in an inclined air layer was reported by Ayyaswamy and Catton (1973):

\[ Nu = Nu_{90} \cdot (\sin \phi)^{1/4} \quad (2.67) \]

It was suggested (Incropera and DeWitt, 1990) that the correlation can be used when it is beyond the critical inclination stated in Hollands et al. (1976).

The correlation (Equation 2.67) is valid under the condition \( \phi^* < \phi < 90^\circ \) where \( \phi^* \) is the critical inclination stated in Hollands et al. (1976). The critical condition that is stated in Hollands et al. (1976) is \( \phi^* = 60^\circ \).
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2.2

Figure 2.5a: Comparison between correlations for free convective heat transfer across inclined air layers for $Ra=10000$ and $AR=100$.

Figure 2.5b: Comparison between correlations for free convective heat transfer across inclined air layers for $Ra=100000$ and $AR=100$.

To choose the appropriate correlation for the natural convection in the cover system between inclination angles 60° to 90°, comparison between the correlations by ElSherbiny et al. (1982) and by Ayyaswamy and Catton (1973) was made (Figure 2.5a, 2.5b). It is obvious that the correlation reported by ElSherbiny et al. (1982) predicts Nusselt numbers closer to those by Hollands et al. (1976) for an inclination of 60°. Therefore, correlations by Hollands et al. (1976) for inclination angles between 0° and 60° (Equation 2.62) and ElSherbiny et al. (1982) for inclination angles between 60° and 90° (Equations 2.62 to 2.64) were used for prediction of natural convection heat transfer coefficients within the cover system. Note that the same correlations were
applied to the situation when no air flows through the collector air channel for natural convection between the absorber and cover system.

In order to calculate the transmission-absorptance product \((\tau \alpha)_c\) for use in flat-plate calculations, it is necessary to consider the variation of the beam component with incident angle. This is particularly important when considering the estimation of thermal performance of a solar collector exposed in an outdoor environment. It is also necessary to consider the separate effect of diffuse sky radiation and ground reflection. The correlations that take into account of these variations are reported:

\[
(\tau \alpha) = \frac{I_T(\tau \alpha)_b + I_d(\frac{1 + \cos \phi}{2})(\tau \alpha)_s + \rho I(\frac{1 - \cos \phi}{2})(\tau \alpha)_g}{I_T} \quad \ldots (2.68)
\]

While the transmittance-absorptance product for the beam, and the diffuse radiation from ground reflection and sky can be calculated using the internal function (FUNCTION TALF) in TRNSYS (University of Wisconsin-Madison, 1994) for given incident angles, the effective beam angle for the diffuse radiation and the angle for diffuse radiation from the ground for the calculations can be determined by (Brandemuhl and Beckman, 1980; Klein, 1979):

\[
\theta_b = 59.68 - 0.1388\phi + 0.001497\phi^2 \quad \ldots (2.69)
\]

and

\[
\theta_s = 90 - 0.5788\phi + 0.002693\phi^2 \quad \ldots (2.70)
\]

2.4. Relationship to HWB

To demonstrate the integrity of the model derivation, it is important to conduct variable substitution in the proposed mathematical model. The following variable substitution is suggested such that, when carried out, the proposed mathematical model may be transformed to the existing models discussed previously (Duffie and Beckman, 1974; Parker, 1981) (see section 2.1).

1) \(U_b = 0\), or
2) \(t'_a = t_a\)
2.4.1. Duffie and Beckman (1974)

Duffie and Beckman (1974) assumed a collector of little rear heat loss (high insulation level), the associated mathematical model is somewhat different from that reported by Parker (1981) (which accounts for rear heat loss).

To examine the integrity of the proposed model derivation, the assumption of \( U_b = 0 \) is substitute into Equation 2.40 yields (assuming also for no long-wave radiation exchange between absorber and front ambient environment):

\[
U_L = \frac{K U_r (h_2 h_{r23} + h_2 h_3 + K h_3 h_{r23})}{K(h_2 h_{r23} + h_2 h_3 + K h_3 h_{r23} + h_2 U_r')}
\]

\[
= \frac{U_r (h_2 h_{r23} + h_2 h_3 + K h_3 h_{r23})}{(h_2 h_{r23} + h_2 h_3 + K h_3 h_{r23}) + h_2 U_r'}
\]

\[
U_L = \frac{U_r'}{1 + h_2 U_r'/(h_2 h_{r23} + h_2 h_3 + K h_3 h_{r23})}
\] ...

(2.71)

and

\[
F' = \frac{K(h_3 U_r' + h_3 h_3 + K h_3 h_{r23} + h_2 h_{r23})}{K(U_r' + h_2 + K h_{r23}) h_3 + K h_{r23} (U_r' + h_2)}
\]

\[
= \frac{h_3 U_r' + h_3 h_3 + K h_3 h_{r23} + h_2 h_{r23}}{U_r' h_3 + h_2 h_3 + K h_3 h_{r23} + U_r' h_{r23} + h_2 h_{r23}}
\]

\[
= \frac{1}{1 + U_r' h_{r23} / (U_r' h_3 + h_2 h_3 + K h_3 h_{r23} + h_2 h_{r23})}
\]

\[
F' = \left[ 1 + \frac{U_r' h_{r23}}{(U_r' h_3 + h_2 h_3 + K h_3 h_{r23} + h_2 h_{r23})} \right]^{-1}
\] ...

(2.72)

It can be seen that Equations 2.71 and 2.72 have been transformed to the same format as Equations 2.5 and 2.6.

2.4.2. Parker (1981)
In Parker (1981) approach, equal front and rear ambient temperatures was assumed. Therefore, assuming $t_a^* = t_a$ in the proposed model derivation, it will reduce to the format of Parker's (1981) model.

From Equation 2.39 it is understood that the equivalent surrounding ambient temperature, $t_a^*$, is a function of both the front ($t_a$) and rear ($t_a^*$) ambient temperatures, derived as follow:

$$t_a^* = \frac{U_h P_3}{P_6 P_1} t_a - \frac{P_7}{P_6} t_a$$  \hspace{1cm} (2.73)

When $t_a^* = t_a$,

$$t_a^* = \frac{U_h P_3}{P_6 P_1} t_a - \frac{P_7}{P_6} t_a$$  \hspace{1cm} (2.74)

Simplifying $\frac{U_h P_3}{P_6 P_1}$ yields (see Appendix A7 for meanings of $P$'s),

$$\frac{U_h P_3}{P_6 P_1} = \frac{U_h}{U_i}$$

$$\frac{U_h P_3}{P_6 P_1} = K U_h (h_3 U_i' + h_2 h_3 + K h_r h_{r23} + h_2 h_{r23})$$

$$+ \frac{K (U_i' + U_h) (h_2 h_{r23} + h_2 h_3 + K h_r h_{r23}) + U_i' U_h (h_2 + K h_3) + K h_r (h_2 + K h_3) + K h_r (h_2 + K h_3)}{P_6 P_1}$$  \hspace{1cm} (2.75)

Simplifying $\frac{P_7}{P_6}$ yields (see Appendix A7 for meanings of $P$'s),

$$\frac{P_7}{P_6} = -\frac{[(K U_i' + K^2 h_{r33}) h_3 + K^2 U_i' h_3 + K^2 h_r h_{r33}] h_{r23}}{P_6 P_1}$$

$$+ \frac{[U_i + K h_{r33} + K h_r] U_i' + K^2 h_r h_{r23} + K^2 U_i' h_3 + K^2 U_i' h_3 + K^2 h_r h_{r33}] h_{r23}}{P_6 P_1}$$

$$+ \frac{[(K U_i + K^2 h_{r33} + K U_i') h_2 + K^2 U_i' h_2 + K^2 U_i' h_3 + K^2 h_r h_{r33}] h_{r33}}{P_6 P_1}$$

$$+ \frac{[U_i + K h_{r33} + K h_r] U_i' + K U_i h_3 + K^2 h_r h_{r33}] h_3}{P_6 P_1}$$

$$+ \frac{(K^2 h_r h_{r23} + K U_i h_3) U_i'}{P_6 P_1}$$
\[
\begin{align*}
&= - \{(U_t' + Kh_{r3s}) Kh_2 + (U_t' + Kh_{r3s}) K^2 h_3 \}h_{r23} \\
&\quad + [(U_h + Kh_{r3s} + Kh_t) U_t' + K^2 h_3 h_{r23}, h_2 + K^2 U_t' h_3 h_{r3s}] \\
/l&\{(U_h + Kh_{r3s} + U_t') Kh_2 + K^2 h_3 (U_t' + U_h) + K^3 h_3 h_{r3s}, h_{r23} \\
&\quad + [(U_h + Kh_{r3s} + Kh_t) U_t' + Kh_t (U_h + Kh_{r3s}), h_2 \\
&\quad + K(U_t' + h_3 (Kh_{r3s} + U_h)) \}
\end{align*}
\]

\[
\begin{align*}
&= - \{(U_t' + Kh_{r3s}) (h_2 + Kh_t) Kh_{r23} \\
&\quad + (U_h + Kh_{r3s} + Kh_t) U_t' h_2 + K^2 h_3 h_{r23} h_2 + K^2 U_t' h_3 h_{r3s} \} \\
/l&\{(Kh_{r3s} + U_t') Kh_2 + K^2 h_3 (U_t' + U_h) + K^3 h_3 h_{r3s}, h_{r23} \\
&\quad + (U_h + Kh_{r3s} + Kh_t) U_t' h_2 + Kh_t h_2 (U_h + Kh_{r3s}), h_2 \\
&\quad + K(U_t' + h_3 (Kh_{r3s} + U_h)) \}
\end{align*}
\]

\[
\begin{align*}
&= -[(U_t' + Kh_{r3s}) (Kh_2 h_{r23} + K^2 h_3 h_{r23} h_2 + K^2 U_t' h_3 h_{r3s}) \\
&\quad + Kh_2 h_2 (U_t' + Kh_{r3s}) + K^2 U_t' h_3 h_{r3s}] \\
/l&\{(Kh_{r3s} + U_t') (Kh_2 h_{r23} + K^2 h_3 h_{r23} + Kh_2 h_3) + U_t' U_h (h_2 + Kh_t) \\
&\quad + K^2 h_2 h_{r23} h_{r3s} + K^3 h_3 h_{r23} h_{r3s} + K(U_t' h_3 h_{r3s} + K^2 h_3 h_{r3s} \\
&\quad + K^2 U_t' h_3 h_{r3s}) \}
\end{align*}
\]

\[
\begin{align*}
&= -[(U_t' + Kh_{r3s}) (Kh_2 h_{r23} + K^2 h_3 h_{r23} h_2 + K^2 U_t' h_3 h_{r3s}) + U_t' h_2 (U_h + Kh_{r3s}) \\
&\quad + K^2 U_t' h_3 h_{r3s}] / \{(Kh_{r3s} + U_t') (Kh_2 h_{r23} + K^2 h_3 h_{r23} + Kh_2 h_3) \\
&\quad + U_t' U_h (h_2 + Kh_t) + Kh_{r3s} [Kh_2 h_{r23} + K^2 h_3 h_{r23} + U_t' h_2 \\
&\quad + Kh_2 h_3 + K(U_t' h_3)] \}
\end{align*}
\]

\[
\frac{P_s}{P_b} = -[(U_t' + Kh_{r3s}) (Kh_2 h_{r23} + K^2 h_3 h_{r23} h_2 + K^2 U_t' h_3 h_{r3s}) + U_t' h_2 (U_h + Kh_{r3s}) \\
&\quad + K^2 U_t' h_3 h_{r3s}] / \{(U_t' + U_h) (Kh_2 h_{r23} + K^2 h_3 h_{r23} + Kh_2 h_3) \\
&\quad + U_t' U_h (h_2 + Kh_t) + Kh_{r3s} [Kh_2 h_{r23} (h_2 + Kh_t) + U_t' (h_2 + Kh_t) \\
&\quad + Kh_2 h_3)] \}
\]

\text{(2.76) }

\text{Therefore,}
\[
\frac{U_b P_3}{P_6 P_1} + \frac{P_7}{P_6} = KU_b (h_3 U_h' + h_2 h_3 + Kh_2 h_{23} + h_2 h_{23})
\]

\[
\begin{align*}
&/\{K(U_h' + U_b)(h_2 h_{23} + h_2 h_3 + Kh_2 h_{23}) + U_h' (h_2 + Kh_3) \\
&+ Kh_{23}[U_h'(h_2 + Kh_3) + Kh_{23}(h_2 + Kh_3) + Kh_2 h_3]\} \\
&+\{(U_h' + +Kh_{23})(Kh_2 h_{23} + K^2 h_2 h_{23} + Kh_2 h_3) \\
&+U_h' h_2 (U_h + Kh_2 h_{23} + U^2 U_h' h_2 h_{23})\} \\
&/\{(U_h' + +U_b)(Kh_2 h_{23} + K^2 h_2 h_{23} + Kh_2 h_3) + U_h' (h_2 + Kh_3) \\
&+Kh_{23}[Kh_{23}(h_2 + Kh_3) + U_h'(h_2 + Kh_3) + Kh_2 h_3]\} \\
\end{align*}
\]

\[
(Kh_2 U_h + U_h + U_b (Kh_2 h_3 + K^2 h_2 h_{23} + Kh_2 h_{23}) \\
+(U_h' + +Kh_{23})(Kh_2 h_{23} + K^2 h_2 h_{23} + Kh_2 h_3) \\
+U_b U_h' h_2 + Ku_h'(h_2 h_3 + U^2 U_h' h_2 h_3)\} \\
/\{(U_h' + +U_b)(Kh_2 h_{23} + K^2 h_2 h_{23} + Kh_2 h_3) + U_h' (h_2 + Kh_3) \\
+Kh_{23}[Kh_{23}(h_2 + Kh_3) + U_h'(h_2 + Kh_3) + Kh_2 h_3]\} \\
\]

\[
= [U_h' + U_b (h_2 + Kh_3) + Ku_h (h_2 h_3 + Kh_2 h_{23} + h_2 h_{23}) \\
+K(u_h' + +Kh_{23})(h_2 h_{23} + Kh_2 h_{23} + h_2 h_3) \\
+Ku_h' h_2 h_3 + K^2 u_h' h_2 h_{23}\} \\
/\{(U_h' + +U_b)(Kh_2 h_{23} + K^2 h_2 h_{23} + Kh_2 h_3) + U_h' (h_2 + Kh_3) \\
+Kh_{23}[Kh_{23}(h_2 + Kh_3) + U_h'(h_2 + Kh_3) + Kh_2 h_3]\} \\
\]

\[
= [K(u_h' + +U_b)(h_2 h_3 + Kh_2 h_{23} + h_2 h_{23}) + U_h' (h_2 + Kh_3) \\
+K^2 Kh_{23}(h_2 h_{23} + Kh_2 h_{23} + h_2 h_3) + Ku_h' h_2 h_3 + K^2 Ku_h' h_2 h_{23}\} \\
/\{(K(u_h' + +U_b)(h_2 h_{23} + Kh_2 h_{23} + h_2 h_3) + U_h' (h_2 + Kh_3) \\
+Kh_{23}[Kh_{23}(h_2 + Kh_3) + U_h'(h_2 + Kh_3) + Kh_2 h_3]\} \\
\]

\[
= \{K(u_h' + +U_b)(h_2 h_3 + Kh_2 h_{23} + h_2 h_{23}) + U_h' (h_2 + Kh_3) \\
+Kh_{23}[Kh_{23}(h_2 + Kh_3) + K^2 h_2 h_{23} + Kh_2 h_3 + U_h' (h_2 + Ku_h' h_3)\} \\
/\{K(u_h' + +U_b)(h_2 h_{23} + Kh_2 h_{23} + h_2 h_3) + U_h' (h_2 + Kh_3) \\
+Kh_{23}[Kh_{23}(h_2 + Kh_3) + U_h'(h_2 + Ku_h' h_3) + Kh_2 h_3]\} \\
\]
\[
\begin{align*}
\frac{U_b P_1}{P_6 P_7} + \frac{P_7}{P_6} &= 1
\end{align*}
\]...

This means that when \( t_{a}' = t_a \), \( t_{a}^* = t_a \). This implies that when the proposed model is used for the condition when the front and rear temperature are equal \( (t_{a}' = t_a) \) (and assuming no long-wave radiation loss between absorber and front environment), the proposed model will reduce to that of Parker (1981).

### 2.5. Summary

A mathematical model of a building-integrated flat-plate air-heating solar collector, with a profiled-metal absorber and 3-layered cover system, has been presented. The model presents an extension of previous work in this field by being able to account for differing ambient temperatures at the front and rear environments of a collector. This is directly relevant to the analysis of multi-functional integrated building facades.

An equivalent surrounding ambient temperature, \( t_{a}^* \), has been defined. This is the temperature at which a collector, when subjected to this temperature \( (t_{a}^*) \) at front and rear, will give the same thermal performance as for the condition when front \( (t_a) \) and rear \( (t_{a}') \) ambient temperatures differ. This newly-created temperature term will enable thermal performance comparisons to be made between a collector which is subjected to different front and rear ambient temperatures \( (t_a \neq t_{a}') \) (such as a building-integrated unit) and a collector which is considered to have front and rear ambient temperatures as equal \( (t_a = t_{a}') \).

The integrity of the derivation of the proposed mathematical model has been tested. Two variables were used. The model was first tested by eliminating the rear heat loss coefficient \( (U_{r} = 0) \) (and assuming no long-wave radiation transfer between the absorber and collect front ambient environment), the proposed mode was reduced to that derived by Duffie and Beckman (1974). The model was then tested by assuming equal front and rear ambient...
temperature \( t_a' = t_a \) (and no long-wave radiation transfer between the absorber and collector front ambient environment), the proposed model was then reduced to that derived by Parker (1981). These demonstrations proved that the derivation of the proposed model is genuine.

Expressions for the collector overall heat loss coefficient, \( U_L \), and for the efficiency factor, \( F' \), have also been derived. Note that if the area enhancement factor becomes unity, and the IR radiation heat transfer coefficient between the absorber and sky becomes zero, then both \( U_L \) and \( F' \) would reduce to those reported by Duffie and Beckman (1991) for a simple flat-plate air-heating solar collector. Both \( F' \) and \( U_L \) are used during validation of the mathematical model since front and rear ambient temperatures were equal.

Expressions for predicting the surface temperatures give additional indices for comparison with those measured during validation. Predictions of temperature are also necessary in the simulation phase of the work (see Chapter 6).

Expressions for predicting the heat transfer coefficients were taken from various literature. Heat transfer correlations for the natural convection mechanism within the multi-layered cover system were selected.
Nomenclature

\( A \) = collector surface area \((m^2)\)
\( A_e \) = area of edge plate \((1\text{-side})\) \((m^2)\)
\( A_x \) = cross-sectional area of the air channel \((m^2)\)
\( AR \) = aspect ratio (-)
\( d_2 \) = thickness of rear insulation \((m)\)
\( d_t \) = thickness of edge plate insulation \((m)\)
\( dh \) = thickness of rear insulation \((m)\)
\( dh \) = hydraulic diameter \((m)\)
\( \Delta t_f \) = change of fluid temperature per metre of channel length \((°C m^{-1})\)
\( F'\) = collector efficiency factor (-)
\( F_R \) = heat removal factor (-)
\( h_1 \) = convection heat transfer coefficient between cover outer surface and front ambient environment \((Wm^{-2}°C^{-1})\)
\( h_2 \) = convection heat transfer coefficient between channel fluid and the cover surface \((Wm^{-2}°C^{-1})\)
\( h_3 \) = convection heat transfer coefficient between channel fluid and the absorber surface \((Wm^{-2}°C^{-1})\)
\( h_4 \) = convection heat transfer coefficient between rear surface to rear ambient environment \((Wm^{-2}°C^{-1})\)
\( h_{c11a} \) = natural convection heat transfer coefficient between the top and middle layers of the cover system \((Wm^{-2}°C^{-1})\)
\( h_{c102} \) = natural convection heat transfer coefficient between the middle and bottom layers of the cover system \((Wm^{-2}°C^{-1})\)
\( h_{r1} \) = radiation heat transfer coefficient between cover outer surface and sky \((Wm^{-2}°C^{-1})\)
\( h_{r11a} \) = radiation heat transfer coefficient between the top and middle layers of the cover system \((Wm^{-2}°C^{-1})\)
\( h_{r102} \) = radiation heat transfer coefficient between the middle and bottom layers of the cover system \((Wm^{-2}°C^{-1})\)
\( h_{r23} \) = radiation heat transfer coefficient between absorber and cover surface that forms part of the flow channel \((Wm^{-2}°C^{-1})\)
\( h_{r34} \) = IR radiation heat transfer coefficient between absorber and sky \((Wm^{-2}°C^{-1})\)
\( h_{r4} \) = radiation heat transfer coefficient between rear surface and rear environment \((Wm^{-2}°C^{-1})\)
\( I \) = total horizontal irradiance on the cover \((Wm^{-2})\)
\( I_b \) = incident beam irradiance on the cover \((Wm^{-2})\)
\( I_T \) = total incident irradiance on the cover \((Wm^{-2})\)
\( I_d \) = horizontal diffuse irradiance on the cover \((Wm^{-2})\)
\( k \) = conductivity of air \((Wm^{-1}°C^{-1})\)
\( k_2 \) = conductivity of rear insulation \((Wm^{-1}°C^{-1})\)
\( k_e \) = conductivity of edge plate insulation \((Wm^{-1}°C^{-1})\)
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- **$K$** = area enhancement factor for corrugation (-)
  
  = (absorber area) / (cover area)

- **$K_{cover}$** = equivalent conduction heat transfer coefficient for the cover system (Wm$^{-2}$°C$^{-1}$)

- **$L$** = length of collector (m)

- **$L'$** = characteristic length (for $h_L$)

- **$L''$** = mean of length and width (m)

- **$L''_c$** = cube root of house volume (m)

- **$m$** = mass flow rate of channel fluid (kgs$^{-1}$)

- **$n$** = a parameter (-)

- **$P_{i,2,...}$** = intermediate variables (see Appendix A7)

- **$Pr$** = Prandtl number (-)

- **$P_w$** = wetted perimeter of the air channel (m)

- **$q_u$** = utilised heat per unit area from the collector (Wm$^{-2}$)

- **$Q_u$** = utilised heat from the collector (W)

- **$Ra$** = Rayleigh number (-)

- **$Re$** = Reynolds number (-)

- **$S$** = energy transferred to absorber (W)

- **$t_a$** = front ambient temperature (°C)

- **$t_a'$** = rear ambient temperature (°C)

- **$t_a^*$** = equivalent ambient temperature (°C)

- **$t_f$** = fluid temperature in fluid channel (°C)

- **$t_i$** = inlet temperature (°C)

- **$t_o$** = outlet temperature (°C)

- **$t_m$n** = number of hours from midnight (h)

- **$T_1$** = cover front surface temperature (°C)

- **$T_2$** = cover surface that forms part of the flow channel (°C)

- **$T_3$** = absorber surface temperature (°C)

- **$T_4$** = rear surface temperature (°C)

- **$T_a$** = ambient temperature (K)

- **$T_dp$** = dew point temperature (°C)

- **$T_p$** = absorber plate temperature (°C)

- **$T_s$** = sky temperature (K)

- **$U_{1}$** = heat transfer coefficient for edge plate (Wm$^{-2}$°C$^{-1}$)

- **$U_{1}$** = front heat transfer coefficient from rear cover surface to front ambient environment (Wm$^{-2}$°C$^{-1}$)

- **$U_{1}'$** = front heat transfer coefficient from rear cover surface to front ambient environment (also account for edge loss) (Wm$^{-2}$°C$^{-1}$)

- **$U_{h}$** = heat transfer coefficient from absorber surface to rear ambient environment (Wm$^{-2}$°C$^{-1}$)

- **$U_{L}$** = overall heat loss coefficient (Wm$^{-2}$°C$^{-1}$)

- **$V_{r}$** = cover surface or meteorological wind speed (ms$^{-1}$)

- **$V_{r}'$** = rear surface wind speed (ms$^{-1}$)

- **$W$** = width of collector module (m)
$x$ = distance normal to the flow direction from the element considered (m)

$y$ = distance in the flow direction from collector inlet to the element considered (m)

$\sigma$ = Stefan-Boltzmann Constant

\[
\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{°C}^{-4}
\]

$\alpha$ = absorptance of the absorber (-)

$\varepsilon_c$ = emissivity of cover layers (-)

$\varepsilon_3$ = emissivity of absorber surface [fluid channel side] (-)

$\varepsilon_d$ = rear surface emissivity (-)

$\phi^*$ = slope of the collector (°)

$\phi^c$ = critical slope of the collector (°)

$\theta$ = angle of the vees for a vee-corrugated absorber (°)

$\tau$ = transmittance of the cover (-)

$\rho$ = density of air in the solar collector (kgm$^{-3}$)

$(\tau\alpha)_e$ = overall transmittance-absorbance product for collector system (-)

$(\tau\alpha)_c$ = effective transmittance-absorbance product (-)

\[
(\tau\alpha)_c = 1.02 \times (\tau\alpha)
\]

$(\tau\alpha)_h$ = transmittance-absorbance product for beam component of irradiance on collector system (-)

$(\tau\alpha)_g$ = transmittance-absorbance product for diffused irradiance due to ground reflection (-)

$(\tau\alpha)_s$ = transmittance-absorbance product for diffused irradiance due to cloud scattering (-)

$\rho_g$ = ground reflectance (-)

$\eta$ = collector efficiency (-)

$\phi_e$ = effective beam incident angle for diffuse irradiance from the ground (°)

$\phi_s$ = effective beam incident angle for diffuse irradiance from the sky (°)
References

Ariel Plastic Ltd (1990), 'Technical Specification: Ariel Corotherm High Insulated Roofing Polycarbonate Twin & Triple Wall Sheets'


Chapter 2 - Mathematical modelling of the structurally-integrated air heating solar collector


Chapter 2 - Mathematical modelling of the structurally-integrated air heating solar collector


Chapter 3 - Laboratory testing of the proposed air heating solar collector

3.1. Introduction

The measurement of solar collector performance provides a means for evaluating the thermal behaviour of a solar collector, as well as a means for providing data for design purposes. Such data can also be used for validation purposes of standard HWB equation. Measurements are usually performed under steady or quasi-steady conditions, either indoors or outdoors. Although the measurement of solar collector performances in the outdoor environment is often the most realistic and practical situation, collector parameters depend on variables such as wind speed, ambient temperature, solar incidence angle, and level of irradiance; these vary in magnitude, and, it is difficult to ensure that the collector reaches thermal equilibrium. Various methods (Hotchkiss et al., 1985; Wang et al., 1987; Emery and Roger, 1984) have been derived for determining the quasi-steady state thermal performance of solar collectors from outdoor tests, but they are complex and time consuming.

The use of a solar simulator for the evaluation of thermal performance permits collector testing to take place indoors under standard conditions of wind, ambient temperature, flow rate and simulated solar irradiance. An evaluation of 23 collectors (Simon, 1976) comprised of different absorber materials, absorber coatings and glazing materials concluded that thermal performances determined using a solar simulator are in good agreement with those determined under outdoor conditions. The British Standard Method of Test for Thermal Performance of Solar Collectors (British Standards Institution, 1986) confirms this statement, performances having been found to agree to within 3% of those found in outdoor tests for a range of flat plate collectors.

For this study, it was decided to carry out performance measurements for the proposed collector geometry within a controlled environment in the laboratory. The reasons are as follows:

- It allows the thermal performance of the solar collector to be evaluated for a wide range of different environmental conditions (such as irradiance and ambient temperature, for example) within a reasonable period of time.
Chapter 3 - Laboratory testing of the proposed air heating solar collector

- Individual environmental conditions such as ambient and inlet air temperatures can be controlled with reasonable accuracy. This is particularly important for air-heating solar collectors because the relatively small thermal capacitance of the heat transfer medium renders the collector performance to be more sensitive to any changes in the surrounding ambient temperature.

- The measurements can be steady-state, and can be carried out with commercially available instrumentation to an accuracy of approximately 10% \((Kreider and Kreith, 1981)\). Performances obtained by the steady-state method have been found to agree to within 3% of those obtained by transient (outdoor) test methods for a range of flat-plate collectors (both liquid and air) \((British Standards Institution, 1986)\). Therefore, indoor measurement offer acceptably realistic results as well as repeatability of test conditions.

- Although solar collectors, when exposed outdoors, are often subjected to highly intermittent solar radiation, a relatively small discrepancy in efficiency was observed (less than 1.5%) between a transient model and a zero-capacitance model for air-heating solar collectors \((Yusoff and Close, 1979)\).

- Since the determination of solar collector thermal performance (in terms of collector heat removal factor \((F_R)\) and overall heat loss coefficient \((U_L)\)) requires measurements of the inlet / ambient temperature difference \((t_i - t_a)\) and outlet / inlet temperature difference \((t_o - t_i)\) at different inlet fluid temperatures, it is necessary to vary the inlet fluid temperature while other variables remain constant during a test. Indoor steady-state testing allows individual variables to be varied separately, making it relatively less time consuming compared with testing to generate performance results from outdoor measurements.

The 'steady-state' thermal performance of a solar collector may be represented by the linear Hottel-Whillier-Bliss (HWB) equation as:

\[
\eta = F_R \tau_0 + F_R U_L \frac{t_i - t_o}{t_i}
\]  
...(3.1)
As can be seen, the efficiency is related to inlet air and ambient temperatures and the irradiance on the collector. It is also related to the outlet / inlet temperature difference, as described in the following equation:

\[ \eta = \frac{mC_p(t_o - t_i)}{I} \quad \text{...(3.2)} \]

It can be seen from Equations 3.1 and 3.2 that it is possible to determine the coefficients \( F_r \) and \( U_L \) when the inlet \( t_i \) and outlet \( t_o \) air temperatures, ambient air temperature \( t_a \), solar irradiance \( I \), effective transmittance-absorptance product \( (\tau \alpha) \), mass flow rate of air \( m \) and specific heat capacitance of air \( C_p \) are known. In a laboratory environment where the simulated solar irradiance is known and the ambient air temperature is known and controlled, the collector performance in terms of \( F_r \) and \( U_L \) can be determined simply by measuring the mass flow rate of air \( m \), outlet-inlet temperature difference \( t_o - t_i \) and inlet-ambient temperature difference \( t_i - t_a \) at different inlet temperatures.

Various standards for solar collector testing have been proposed in various parts of the world. In this study, the method of test for the proposed solar collector was broadly based on that of BS 6757, British Standard Methods of Test for Thermal Performance of Solar Collectors (British Standards Institution, 1986) and that of the ASHRAE Standard Methods of Testing to Determine the Thermal Performance of Solar Collectors (ASHRAE, 1986). Note, however, that the aim was to obtain general performance data, and to use it for model validation purposes, rather than to conduct tests which rigorously adhere to the standard laid down.

### 3.2. Design of simulator and method of measurement

A specially-designed indoor solar simulator was constructed to enable the thermal performance of the proposed solar collector to be measured. The arrangement consisted of the following main components:

1) a vertical section of the covered profiled steel collector, housed within a tubular metal framework; air entered the collector at the base and was removed at the top;
2) a set of four Thorn 1000 Watt compact source iodide lamps, forming a square array of variable dimensions; the array was fixed to a wheeled base which allowed movement towards or away from the test surface.

3) the entire arrangement was accommodated within an air-conditioned, high-ceiling laboratory, the walls of which contained insulation so as to provide a uniform radiant temperature.

Details of the system components, together with the measurement techniques employed, are given in the following sections.

3.2.1. Solar collector

This consisted of a 2 m long section of British Steel 'long rib 1000' profiled cladding with a profile depth of 0.03 m (trough to crest). The steel surface was finished with a black 'Pvf2' coating, capable of withstanding a surface temperature of 120 °C (British Steel, 1992). Both ends of the profiled cladding were extended and formed part of the inlet and outlet ducts; this was carried out so as to assist the flow development of the incoming air flow to the collector, as if the test section were part of a large profiled facade. A backing of 0.1 m thick fibreglass insulation was attached to the rear of the steel profile, this being consistent with the typical thickness found in practice for this type of construction. A metal plate provided a rear surface finish. A transparent cover system, consisting of triple-layered polycarbonate rigid sheeting ('Corotherm') with a layer separation of 7 mm was attached to the front of the collector. A spacing of 0.1 m was left between the underside of the cover and the top surface (crest) of the profile absorber, thus forming the collector air flow duct. 'Corotherm' sheeting is a common building material for greenhouse and conservatories, its multi-layer construction offering improved insulation compared with conventional polycarbonate sheeting. It is recommended by the manufacturers (Ariel Plastic Limited, 1990) as a potential transparent cover for solar collectors. The entire collector test section was constructed in the vertical position so as to represent a section of a wall. Figures 3.1a and 3.1b illustrate the arrangement.
3.2.2. Collector air flow and measurement

Air from the laboratory initially entered the flow system via a round duct of 300mm diameter. The inlet air temperature was adjusted by an electric heater, power to which was controlled and was supplied by a variac. A series of metal meshes promoted mixing of the pre-heated air. In order to reduce any flow disturbances caused by the air heater and mixer, a honeycomb air flow straightener was inserted in the circular duct section. The pre-heated air was then passed through a rectangular inlet air plenum and, in turn, entered the collector. The air (the collector heat transfer medium) was heated by passage over the absorber. Upon exiting the collector, the outlet air then passed through a rectangular plenum and then through a round outlet duct. A fan attached at the end of the round outlet duct was used to induce the desired air mass flow rate. The air was finally expelled to the laboratory once again.

The air mass flow rates at the entrance to and exit from the collector were measured at positions 150mm before the inlet and 150mm after the outlet of the collector test section. A hot wire anemometer was used for this purpose, having an accuracy of ±0.1 m/s for velocities below 2 m/s, and ±0.2 m/s for velocities between 2 and 5 m/s. At each location (inlet and outlet), ten re-
sealable access points were drilled for insertion of the hot-wire anemometer so as to allow traverses to be performed. At each location, measurements were conducted so that a grid (or matrix) consisting of 72 values of velocity measurements was obtained at each cross-section for inlet and outlet. The total volume flow rate could then be determined using the following expression:

\[ V = \sum_{r=1}^{n} A_r \cdot v_r \]  

...(3.3)

where

- \( A_r \) = cross-sectional area at a given measurement point (m²)
- \( v_r \) = velocity at that measurement point (ms⁻¹)
- \( V \) = volumetric flow rate (m³s⁻¹)
- \( n \) = total number of measurement points in the matrix (-)

The above measurement technique can detect the presence of any significant air leakage into or out of the collector test section.

However, accurate assessment of air leakage subsequently proved to be difficult, due to the overall experimental uncertainty of the measurement process (see later).

3.2.3. Wind simulation

The presence of a wind flow at the front of the collector surface is important, as it provides a known convective condition, and is therefore recommended in collector testing (British Standards Institution, 1986).

A simulated surface wind over the front surface of the collector cover was generated by three direct-current axial fans. They were arranged in linear formation at the base of the collector near the inlet. The mean surface wind speed across the collector face was determined from measurements with a hot-wire anemometer; a 'grid' array of 4 by 4 measuring points (each 250 mm by 250mm) was used, and the mean value of wind speed then calculated by simple averaging. The air velocity was measured before each test at each point, at a height of 5 mm above the cover. The average surface wind speed was found to be 3.5 ms⁻¹ for all tests.
3.2.4. Temperature measurements

For accurate determination of the collector performance as described by Equations 3.1 and 3.2, the most important temperature measurements required are the differences between outlet and inlet \((t_o - t_i)\) and between inlet and the surrounding ambient air \((t_i - t_a)\). Special arrangements were therefore made to obtain these readings, as follows.

**Outlet-inlet air temperature difference**

The average temperature difference across the collector test section was measured using a calibrated thermopile formed by series-connected thermocouples. These were a total of 6 'cold' and 6 'hot' junctions. The 'cold' junctions of the thermopile arrangement were placed in the inlet air stream (100mm before the collector inlet). Similarly, the 'hot' junctions were placed in the outlet air stream (100mm after the collector outlet). Each junction was protected from direct simulated irradiance by covering with aluminium the entrance and exit region (extended for flow development purposes) of the collector (see Figure 3.1b), leaving only the 1m² centre area exposed to the simulated solar irradiance (the collector test section). Individual thermocouple junctions were, in addition, radiation-shielded by aluminium-foil-covered tubing of 300mm in length. The thermopile was calibrated using a controlled-temperature water bath, and the overall sensitivity of the thermopile arrangement was found to be 0.2368 mV°C⁻¹. The thermocouple junctions were evenly distributed across outlet and inlet sections.

**Inlet-ambient temperature difference**

Another calibrated thermopile arrangement was made for measuring the average temperature difference between the inlet air and the surrounding laboratory ambient air. The six 'hot' junctions were distributed evenly across the inlet air stream; the six 'cold' junctions (for measuring the surrounding ambient air temperature) were located as follows: three were located in front of the collector air outlet plenum, such that the air temperature of the simulated wind flow across the collector face could be measured as the wind flow departed from the collector; two were located near the collector edge plates (one each side); and one was located near the rear side of the
collector. All junctions were radiation shielded. The overall sensitivity of the thermopile was found by calibration to be 0.2403 mV°C⁻¹.

**Absorber surface temperature**

Two sets of parallel-connected thermocouples were used for measuring the average temperature of the profiled absorber surface. The first set consisted of 5 parallel-connected thermocouples, each junction being secured 150mm beyond the collector inlet in the 'troughs' of the profiled absorber. The second set consisted of 6 parallel-connected thermocouples, each junction being secured 150mm before the collector outlet on the 'crests' of the profiled absorber. In this way, the mean surface temperatures of the absorber 'troughs' and 'crests' could be compared, and the overall average absorber surface temperature could be measured. Heat sink paste was applied at each thermocouple junction attachment point, to maximise the thermal bonding to the absorber surface. Black tape was placed on top of each junction so as to produce a measuring point of the same emissivity as that of the surface being measured (black Pvf2). It was not possible to embed the thermocouple junctions within the absorber by drilling from the underside because of the thinness of the absorber and difficulties of access.

**Cover surface temperatures**

Similar arrangements were made for measuring the polycarbonate cover inner and outer surface temperatures. A set of 10 parallel-connected thermocouples were used for measuring the cover inner surface, and a set of 4 parallel-connected thermocouples were used for measuring the cover outer surface. Heat sink paste was again applied between each thermocouple junction and its surface attachment point, the junction finally being covered with transparent tape.

**Edge plate and ambient temperatures**

Individual thermocouples were situated on the extended surface of the edge plate used for enclosing the sides of the collector flow channel. In this way, side heat losses from the collector flow channel could be estimated. A thermocouple at the collector inlet was also used to monitor the actual temperature of the air entering the collector.
Solar irradiance

The solar simulator consisted of four 1000W Compact Source Iodide (CSI) lamps, arranged in a squared array. These lamp types have been reported to give a good approximation to the solar spectrum, and have been used extensively in solar simulator work (Krusi and Schmid, 1983). Individual lamp positions within the array could be adjusted to obtain the best irradiance uniformity on the 1 m² aperture of the solar collector test section for a particular lamp-collector distance. In order to achieve this, both the lamp position and the simulator-collector distance had to be adjusted.

The actual irradiance distribution on the cover surface was checked by measurement using an Eppley pyranometer. The pyranometer was attached to a framework fitted across the collector face, and measurements of irradiance were made across a 'gridwork' of 100mm square grid size, as suggested by the British Standard (British Standards Institution, 1986). An average irradiance of 406.7 Wm⁻² was measured on the 1 m² test area, for a lamp-to-lamp spacing of 0.84 m and lamp-collector distance of 4 m. The coefficient of variation (CV), defined as the ratio of the standard deviation to the mean, was found to be 15.8%. Figure 3.2 shows the irradiance distribution.

Figure 3.2: Irradiance distribution for a collector-simulator distance of 4m
For a CSI lamp, 90% of the energy from the lamp is contained within a subtended angle 12°, 95% within approximately 15° (Krusi and Schmid, 1983); this does meet the requirement of the ASHRAE Standards (1986) of 90% within a subtended angle of 20°.

When testing solar collectors in accordance with existing standards, the requirement on irradiance uniformity according to the ASHRAE Standard is that the variations should not vary from the average value by more than ±3%. The output of the solar simulator was 406.7 Wm⁻² ± 64.3 Wm⁻². This corresponds to a 12.5% variation. The simulator designed for this study does not meet this requirement, because the purpose of this part of the investigation was for model validation, together with an estimate of likely collector performance for design purposes.

3.3. Measurement of cover and absorber properties

In order to determine values for the heat removal factor \( (F_h) \) and the overall heat loss coefficient \( (U_1) \) of the solar collector, and also to enable the solar collector to be modelled with accuracy, the optical properties of the absorber and cover need to be determined. This can be done by testing samples of both the absorber and cover in a spectrometer. Measurements of transmission and absorption over a range of incident wavelengths were made. It was then necessary to 'weight' these properties with respect to the particular spectrum that is incident (ASTM, 1979), i.e. whether it is solar radiation, or radiation from CSI lamps.

The spectrometer used was the Bruker IFS 66/S integrating-sphere spectrometer at Oxford Brookes University. Two samples of absorber material were tested, one black and one green; both were standard Pvf2 finishes offered by their manufacturer. The spectral transmissivity and reflectivity measurements employ the integrating-sphere technique (ASTM, 1982). The reflectivities of the absorber samples were measured at wavelengths between 0.4 μm and 25 μm (for black absorber) and between 0.4 μm and 0.8 μm (for green absorber). Since the absorber materials are opaque, the absorptivities \( (\alpha) \) could be obtained from:

\[
\alpha = 1 - \rho
\]
where

\[ \alpha = \text{absorptivity} \]  
\[ \rho = \text{reflectivity} \]  

Figure 3.3: Absorptivity measurement of the absorber (with solar simulator spectral output)

Figure 3.3 shows the absorptivity measurements and the spectral output of the solar simulator lamps. The spectral output from the solar simulator lamps is reported over the range 0.4 \( \mu \)m to 0.8 \( \mu \)m (the main fraction of their energy output); the mean absorptivity was therefore calculated over this range of wavelengths. The mean absorptivity was calculated by weighting the spectral absorptivity with the spectral output of the solar simulator CSI lamps (ASTM, 1979). The mean absorptivity of the black Pvf2 finish to CSI radiation was calculated to be 0.96.

One sample of the triple-layer polycarbonate cover was prepared for testing. For the cover material, the spectral transmissivity at wavelengths between 0.4 \( \mu \)m and 0.8 \( \mu \)m was measured and is presented in Figure 3.4. Since the three horizontal layers of the cover are supported by vertical 'bridges' (also of polycarbonate), the transmissivities through both the 'clear' layers and the 'bridges' were measured. The transmissivity of both the 'clear' layers and the 'bridges' were again calculated by weighting the spectral transmissivity with reference to the spectral output of the solar simulator CSI lamps over the range 0.4 \( \mu \)m to 0.8 \( \mu \)m. The overall transmissivity of the cover system was then determined by averaging according to the area ratio of the 'clear' layers and the 'bridges'. The mean transmissivity of the triple-layer polycarbonate cover to CSI radiation was calculated to be 0.68.
Kirchhoff's Law shows that, for a given temperature, the total hemispherical emissivity of a grey surface absorptivity (Simonson, 1984), i.e. $\varepsilon_\lambda = \alpha_\lambda$.

where

\[
\begin{align*}
\alpha_\lambda & = \text{spectral absorptivity at wavelength } \lambda \text{ (-)} \\
\varepsilon_\lambda & = \text{spectral emissivity at wavelength } \lambda \text{ (-)} \\
\lambda & = \text{wavelength (\text{\mu}m)}
\end{align*}
\]

To determine the emissivity of the materials, their spectral absorptivities over the wavelength range 1.8 \text{ \mu}m to 25 \text{ \mu}m was measured. This was then weighted with the blackbody spectral emissive power at 20°C.

A similar approach was used to obtain the cover emissivity. Here, the reflectivities of the material were measured over the wavelength 1.8 \text{ \mu}m to 25 \text{ \mu}m. The corresponding spectral absorptivities were then calculated. These were then weighted against the blackbody spectral emissive power at 50°C to obtain the corresponding spectral emissivity. The absorptivities for both the absorber and the cover materials are presented in Figures 3.5 and 3.6.

Using the approach described, an emissivity of 0.91 was estimated for the black (Pvf2 finished) absorber at 50°C, and of 0.97 for the polycarbonate cover at 20°C.
3.4. Time constant

The collector time constant is the time required for the fluid leaving a solar collector to attain 63.2% of the change from initial to final steady state, following a step change in irradiance. The time constant of the solar collector test section was required because it determined the time needed during tests to attain thermal equilibrium. In other words, it is the time required between taking successive sets of measurements. It is also important to confirm that the thermal performance of the collector can be treated by a steady-state analysis (see Chapter 2).
The governing equation for the transient behaviour of a non-concentrating solar collector is:

\[
\frac{C_a}{A} \frac{dt_f}{dT} = F_R I (\alpha \alpha) - F_R U_L (t_i - t_o) - \frac{m C_p}{A} (t_o - t_i)
\]  

...(3.4)

where

\[C_a\] = effective specific heat capacity of the solar collector test section (J°C⁻¹)

\[t_f\] = average fluid temperature (°C)

If the solar irradiance I is suddenly changed and held constant, and if \((\alpha \alpha)\), \(U_L\), \(t_o\), \(m\) & \(C_p\) can be considered constant for the transient period, and if the rate of change of the transfer fluid exit temperature with time is related to the rate of change of the transfer fluid average temperature with time by:

\[
\frac{dt_f}{dT} = K' \frac{dt_o}{dT}
\]

...(3.5)

where

\[K' = \frac{m C_p}{F' U_L A} \left[ \frac{F'}{F_R} - 1 \right]
\]

...(3.6)

then Equation 3.4 can be solved to give the exit temperature of the transfer fluid as a function of time, as follows:

\[
\frac{F_R I (\alpha \alpha)e - F_R U_L (t_i - t_o) - (m C_p / A)(t_o - t_i)}{F_R I (\alpha \alpha) - F_R U_L (t_i - t_o) - (m C_p / A)(t_o, \text{initial} - t_i)} = e^{\left(\frac{m C_p}{K' C_a}\right) t}
\]

...(3.7)

The quantity \((K C_a)/(m C_p)\) is known as the 'time constant', and is the time required for the quantity on the left hand side of Equation 3.7 to change from 1 to 0.368, where 0.368 = 1/e.

So, in the absence of irradiance and with the inlet fluid temperature being equal to that of the ambient, Equation 3.7 is reduced to:
The time constant was obtained as follows. While the collector was exposed to the source of simulated solar irradiance, laboratory ambient air was drawn (without pre-heating) through the solar collector. Mass flow rates into and out of the collector were measured using the hot-wire anemometer. A period of 30 minutes was allowed for thermal equilibrium of the solar collector system to be established (confirmed by measurement). Once the thermal equilibrium was established, the solar irradiance simulator was switched off and the data-logger was activated to measure the outlet-inlet air temperature difference \((t_o - t_i)\) in 5-second intervals for 20 minutes.

The results are presented as a graph of \(\ln[(t_o - t_i)/(t_{o,initial} - t_i)]\) versus time (Figure 3.7a and b). The gradient equals \(-1/T\) where \(T\) is the time constant of the collector. Figure 3.7a was plotted using measurements obtained in the first 20 minutes after the solar simulator was switched off. After the first 500 seconds, a cyclic disturbance can be observed; this is considered to be caused by corresponding disturbances to the inlet air temperature to the collector, which in turn is drawn from the laboratory ambient. The primary cause of the disturbances to the laboratory air temperature is thought to be cyclic control of the laboratory air conditioning system. During the first 500 seconds these disturbances are not apparent because of the elevated temperature of the solar collector test section in comparison with the ambient temperature. However, as the collector cooled down, the disturbances become apparent as the effective 'signal to noise' ratio reduces. It was therefore decided to re-plot the results obtained during the first 500 seconds, as shown in Figure 3.7b. The time constant can be found from the gradient of the graphs in Figure 3.7a and b.
Figure 3.7a: Time constant by regression fit using measured data (using all data)

The gradients obtained from both graphs were very close. By regression, it was found to be -0.0034, thus giving a time constant of 294 seconds (4.9 minutes). Thus, the time between taking sets of readings during laboratory tests should be at least 5 minutes. However, in order to ensure that thermal equilibrium was reached, 30 minutes was allowed between readings. In addition, the time constant result justifies the steady-state analysis adopted in Chapter 2 for modelling of the collector-facade system.
3.5. Efficiency characteristics for equal front and rear ambient temperatures

3.5.1. Experimental procedure

Measurement of the thermal performance of the covered profiled steel cladding as an air-heating solar collector was carried out by generating standard efficiency characteristics. The experimental procedure was as follows.

1) The fan for inducing air flow through the collector was activated, and was set to achieve a desired mass flow rate. During these tests, two air mass flow rates were investigated: 0.134 kgs⁻¹ and 0.203 kgs⁻¹; these were selected by considering the fresh air requirements of occupants in a typical profiled metal building (see Chapter 5). The three wind simulator fans were activated.

2) The solar simulator CSI lamps were switched on, and thermal equilibrium was allowed to be establish in the solar collector test section (typically 30 minutes was allowed).

3) The air mass flow rate into and out of the collector test section was then measured using a traverse with a hot-wire anemometer. Measurements of the simulated surface wind speed were also performed.

4) A further 30 minutes was allowed after the air mass flow rates had been measured to ensure thermal equilibrium again.

5) The outlet-inlet air temperature difference and the inlet-ambient air temperature differences were then recorded at 5-second intervals for a duration of 4 minutes.

6) The air inlet temperature to the collector test section was then raised by increasing the power supplied to the air heater via the variac.
7) A further 30 minutes was allowed for the new thermal equilibrium state to be established.

8) Step 5 to 7 were repeated until 8 sets of measurements had been obtained.

3.5.2. Experimental uncertainties

Uncertainties occurred in the measurement of the following variables (Table 3.1).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Mass flow rate (kg/s)</th>
<th>Fluid density (kg/m³)</th>
<th>Mean temperature (°C)</th>
<th>Irradiance (W/m²)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Measured 0.134</td>
<td>1.2</td>
<td>0.8998</td>
<td>406.7</td>
<td>0.456</td>
</tr>
<tr>
<td></td>
<td>Uncertainty ±0.011</td>
<td>±0.0025</td>
<td>±0.000422</td>
<td>±50.8</td>
<td>±0.039</td>
</tr>
<tr>
<td>2</td>
<td>Measured 0.203</td>
<td>1.2</td>
<td>0.7225</td>
<td>406.7</td>
<td>0.440</td>
</tr>
<tr>
<td></td>
<td>Uncertainty ±0.017</td>
<td>±0.0025</td>
<td>±0.000422</td>
<td>±50.8</td>
<td>±0.038</td>
</tr>
<tr>
<td>3</td>
<td>Measured 0.202</td>
<td>1.2</td>
<td>0.6685</td>
<td>406.7</td>
<td>0.392</td>
</tr>
<tr>
<td></td>
<td>Uncertainty ±0.017</td>
<td>±0.0025</td>
<td>±0.000422</td>
<td>±50.8</td>
<td>±0.034</td>
</tr>
</tbody>
</table>

Table 3.1: Uncertainty occurred in measurement (see Appendix A5)

The overall uncertainty in the measurements was estimated, and these are presented as error bars in the efficiency characteristics (Figure 3.8a, b and c).

In addition to the preceding random errors in measured variables, there were also systematic errors present in the form of:

1) fluctuation of the laboratory ambient air temperature, thought to be caused by the control of the laboratory air-conditioning system. This is most likely to affect the rear ambient values of collector inlet temperature, i.e. where \((t_i - t_o)/I\) is rear zero. However, inspection of the efficiency characteristics (Figure 3.8a, b and c) shows little departure from linearity in comparison with subsequent measured points, suggesting that this effect is negligibly small.

2) conduction heat transfer occurred along the longitudinal axis of the profiled absorber, removing heat from the test section under consideration. This was caused by the additional lengths of profile employed for flow development purposes. Furthermore, heat was conducted from the extended region of the enclosing edge plates at the side of the collector as
well as conduction from the side of the absorber edge. These constitute additional heat loss paths, the effects of which are not included within the experimental results. However, these effects are not included in the model presented in Chapter 2. The magnitude of these effects are estimated in the next chapter, and are taken into account during the validation (described in the next chapter).

3.5.3. Results

Results were plotted on a standard graph of efficiency ($\eta$) against $(t_i - t_a) / I$, from which the overall collector heat loss coefficient ($U_L$) and the heat removal factor ($F_R$) could be evaluated.

Three efficiency characteristics were obtained for the test conditions stated, and are presented as Figure 3.8a, b and c. From these characteristics, the values obtained for $U_L$ and $F_R$ are given in Table 3.2; results for $U_L$ and $F_R$ are within the ranges 10 to 12 Wm$^{-2}$C$^{-1}$ ($\pm 2$ Wm$^{-2}$C$^{-1}$) and 0.6 to 0.8 ($\pm 0.1$), respectively.

![Figure 3.8a: Efficiency characteristic for $m=0.134$ kgs$^{-1}$, $I=406.7$ Wm$^{-2}$, $V_p=3.5$ ms$^{-1}$, vertical orientation, $(\tau \alpha)_v=0.68$](image-url)
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Figure 3.8b: Efficiency characteristic for $m=0.203$ kg s$^{-1}$, $I=406.7$ W m$^{-2}$, $V_p=3.5$ m s$^{-1}$, vertical orientation, $(\tau \alpha)_r=0.68$

Figure 3.8c: Efficiency characteristic for $m=0.202$ kg s$^{-1}$, $I=406.7$ W m$^{-2}$, $V_p=3.5$ m s$^{-1}$, vertical orientation, $(\tau \alpha)_r=0.68$

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Uncertainty in efficiency (%)</th>
<th>Measured $U_L$ (W m$^{-2}$)</th>
<th>$F_R$ (-)</th>
<th>Measurement corresponds to -ve efficiency uncertainty</th>
<th>Measured $U_L$ (W m$^{-2}$)</th>
<th>$F_R$ (-)</th>
<th>Measurement corresponds to +ve efficiency uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.64</td>
<td>12.15</td>
<td>0.78</td>
<td>13.40</td>
<td>0.61</td>
<td>11.12</td>
<td>0.74</td>
</tr>
<tr>
<td>2</td>
<td>7.99</td>
<td>10.71</td>
<td>0.82</td>
<td>11.72</td>
<td>0.75</td>
<td>9.85</td>
<td>0.89</td>
</tr>
<tr>
<td>3</td>
<td>8.34</td>
<td>12.64</td>
<td>0.62</td>
<td>13.84</td>
<td>0.57</td>
<td>11.64</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Table 3.2: Estimated uncertainties on efficiency and the resulting $U_L$ and $F_R$. 

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The results provide designers with an estimate of the performance of profiled steel cladding as an air-heating solar collector, when covered with a triple-layer polycarbonate transparent cover and rear-insulated to the standard level. They can be used by designers at or near the stated test conditions. Note that these results relate to a condition of equal front and rear ambient temperatures surrounding the collector. In the following section, the situation of differing front and rear ambient temperatures is considered.

3.6. Efficiency characteristics for differing front and rear ambient temperatures

3.6.1. Modified solar simulator

When a solar collector forms an integral part of a building envelope, the ambient temperatures at the front and rear of the collector will differ in the majority of instances. In a heated building, for example, such differences will be exacerbated at the upper regions of a room or zone, where thermal stratification can cause higher indoor temperatures near ceiling level. Furthermore, the thickness of collector rear insulation is an important factor in determining the extent to which such front / rear ambient temperature differences affect collector performance. In order to examine these effects, the solar simulator was modified and further efficiency characteristics were generated; these additional data can then be used to validated the model presented in Chapter 2 for the condition of differing front / rear ambient temperatures.

The configuration of the modified test simulator is shown in Figure 3.9. Note that the collector rear insulation was completely removed, leaving an air space between the absorber rear surface and the back of the test section. This modification effectively created a metal box at the rear of the collector; holes were drilled into plates at the top and bottom of the box section to allow the passage of tempered air through the box. A hot air blower was attached at the entrance to the box section (at the bottom). Power to the heating elements of the air blower was supplied by a variac, thus allowing control of the air temperature supplied to the box. The box was surrounded externally by fibreglass. In this way, a separate temperature-controlled environment was created for the rear of the collector; this environment was independent of the collector front environment (the main laboratory). Note that the removal of all
insulation from the back of the collector (that is, the rear of the absorber plate is in direct contact with the warmed air in the box section) renders the collector most sensitive to the effects of front / rear ambient temperature differences. This corresponds to a rear thermal conductance \( (k_z / d_z) \) of 50000 Wm\(^{-2}\)°C\(^{-1}\). The reminder of the test section and simulator functioned as in the previous tests (section 3.2).

Figure 3.9: Schematic drawing of the modified simulator arrangement.

3.6.2. Conditions for measurement

An average irradiance of 363.8 Wm\(^{-2}\) was measured on the 1 m\(^2\) collector test area, for a lamp-to-lamp spacing of 0.84 m and a lamp-collector distance of 4 m. The coefficient of variation (CV) was found to be 19.6%.

The average simulated wind speed on the front cover surface was measured to be 3.7 ms\(^{-1}\). The air mass flow rate in the collector was measured to be 0.07 kgs\(^{-1}\). The mean air velocity in the rear box section was measured to be 0.26 ms\(^{-1}\). This corresponds to Reynolds number \((Re)\) of about 6400 for flow through in the rear box.

3.6.3. Measurement procedure
A series of tests was conducted for rear ambient temperatures of 22 °C, 24 °C, 28 °C, 32 °C, 36 °C, 39 °C, 42 °C, 46 °C, 48 °C and 52 °C. This was achieved by varying the heat output of the heating element of the air blower to the rear box section. In each test, the front ambient temperature was fixed at a constant value of 18 °C. The rest of the test procedures were as for the previous tests (section 3.5.1).

3.6.4. Results

Results were plotted as a standard graph of efficiency (\(\eta\)) against \((t_a - t_a') / I\), from which the overall collector heat loss coefficient \((U_L)\) and the heat removal factor \((F_h)\) could be evaluated.

![Graph of Efficiency Characteristic](image)

Figure 3.10a: Efficiency characteristic (based on \(t_{av} = (t_a + t_a') / 2\)) for \(t_a = 18\) °C, \(t_a' = 22\) °C, \(m = 0.07\) kgs\(^{-1}\), \(I = 363.6\) Wm\(^{-2}\), vertical orientation, \((\tau \alpha) = 0.68\)
Figure 3.10b: Efficiency characteristic (based on \( t_{a,m} \) \( \left( t_{a} + t_{a}' \right) / 2 \)) for
\( t_a = 18 \, ^\circ\text{C}, \, t_{a}' = 32 \, ^\circ\text{C}, \, m = 0.07 \, \text{kg} / \text{s}, \, I = 363.6 \, \text{Wm}^{-2}, \) vertical
orientation, \((\tau\alpha)_e = 0.68\)

Figure 3.10c: Efficiency characteristic (based on \( t_{a,m} \) \( \left( t_{a} + t_{a}' \right) / 2 \)) for
\( t_a = 18 \, ^\circ\text{C}, \, t_{a}' = 42 \, ^\circ\text{C}, \, m = 0.07 \, \text{kg} / \text{s}, \, I = 363.6 \, \text{Wm}^{-2}, \) vertical
orientation, \((\tau\alpha)_e = 0.68\)
Figure 3.10d: Efficiency characteristic (based on \( t_{a,av} \) \( = (t_a + t_a') / 2 \)) for 
\( t_a = 18 \) °C, \( t_a' = 52 \) °C, \( m = 0.07 \) kgs\(^{-1}\), \( I = 363.6 \) Wm\(^{-2}\), vertical 
orientation, \( (\tau \alpha)_{e} = 0.68 \)

Ten thermal efficiency characteristics, based on the average ambient 
temperature \( (t_{a,av}) \), were obtained for the test conditions stated above; a
sample of experimental results is presented in Figures 3.10a to d. These plots
present the thermal performance characteristics of the collector at different
rear ambient temperatures \( (t_a') \), and hence a range of front \( / \) rear ambient
temperatures (for a constant front ambient temperature of 18 °C). Note that
\( t_a'' \) or \( t_a' \) can be used in place of \( t_{a,av} \) to re-plot the efficiency characteristics,
where \( t_a'' \) is a U-value weighted surrounding ambient temperature, defined
as:

\[
t_a'' = \frac{U'_i t_a + U_k t_a'}{U'_i + U_h} \quad \text{...(3.9)}
\]

and \( t_a' \) is the equivalent ambient temperature surrounding the collector as
defined in Chapter 2 (Equation 2.39). Use of these ambient temperatures
would result in different values for the gradients and intercepts obtained from
the efficiency characteristics, and hence different values for \( F_R \) and \( U_L \).

Figure 3.11 summarise the relationship between \( U_j \) and \( (t_a' - t_a) \) for the three
surrounding ambient temperatures \( (t_{a,av}, t_a'', \text{ and } t_a') \).
Chapter 3 - Laboratory testing of the proposed air heating solar collector

Figure 3.11: Experimentally-measured values for $U_L$ versus $(t_{a} - t_a)$ for three surrounding ambient temperatures ($t_{a,1}$, $t_{a,2}$, and $t_{a,3}$)

As can be seen from Figure 3.11, $U_L$ decreases when $(t_{a} - t_a)$ increases. This is expected, and is caused by the reduction of heat loss from the collector rear surface to the rear ambient environment, as the temperature at the rear of the collector increases. Furthermore, the differences in $U_L$ produced by the use of differing surrounding ambient temperatures is seen to increase as $(t_{a} - t_a)$ becomes larger; this discrepancy in $U_L$ values can be of the order of 20-25%.

3.7. Summary

The design of a laboratory-based test facility and solar simulator has been described; its purpose is to estimate the performance of covered standard profiled steel cladding as an air-heating solar collector and to provide design data. The data can also be used for validation of the collector model presented in Chapter 2 for the proposed application area.

The spectral optical properties of the triple-layered polycarbonate cover and of the profiled cladding absorber have been measured. It is evident that the cover acts well as a component in a solar collector in that it offers some thermal resistance due to the air gap formed by its three layers, while at the same time having a transmittance in the region of 0.71 (for CSI radiation). The black Pvf2 absorber has an absorptivity of 0.96.
The time constant of the solar collector was found to be 4.9 minutes (294 seconds). This showed that the collector has a low thermal capacitance and that its performance may be treated as steady-state, as suggested by Duffie and Beckman (1991).

Experimental uncertainties have been estimated and their effect on experimental results has been discussed. Their implication for validation is discussed in the next chapter.

Figure 3.8a, b and c present the efficiency characteristics of the collector for several conditions. These can be used by designers to estimate the performance of profiled steel cladding as an air-heating solar collector when covered with a triple-layer polycarbonate cover, and in the vertical position (as for a wall), and rear-insulated to the standard level. Corresponding values for \( U_L \) and \( F_R \) have been given. Measurements have been taken for the condition of front and rear ambient temperature being equal, which is the standard condition for solar collector performance evaluation. The effect on \( U_L \) of increasing the front / rear ambient temperature difference has been determined, together with the influence of the manner in which the overall ambient temperature surrounding the collector is defined. These data will permit validation of the collector model presented in Chapter 2.

In the next chapter, the conditions of front / rear temperature difference and rear insulation level are identified for which the standard Hottel-Whillier-Bliss analysis applies and for which the proposed new model is required. The suitability of the model for the proposed application area is considered.
References


British Steel (1992), 'Colorcoat in Building. A Guide to Architectural Practice', British Steel Strip Products, May


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Yusoff M. and Close D. J. (1979), 'Transient studies of Solar Air Heaters', International Symposium on Solar Energy for Development, Tokyo, Japan, 5-10 February
Chapter 4 - Model testing, inter-model comparison and validation

4.1. Introduction

In Chapter 2, a theoretically-derived mathematical model for the prediction of thermal performance of the proposed solar collector was presented. To test the integrity of the model, validation is carried out by comparison of model predictions with laboratory measured thermal performances (presented in Chapter 3). For a solar collector system, comparison between the measured heat removal factor ($F_R$) and the overall heat loss coefficient ($U_L$) with values estimated from a mathematical model is usually carried out for validation purposes.

The laboratory condition used for the initial validation was that of equal front and rear ambient temperatures ($t_a = t_a'$); in this way, it was possible to directly compare predictions of the proposed model with those from the standard Hottel-Whillier-Bliss (HWB) analysis. Here, the theoretically-derived model for this study (Chapter 2) and a standard model for the case of a collector having a triple-layered cover with flow between the cover and an absorber (as reported by Duffie and Beckman, 1991) were compared with one another and against the laboratory measured thermal performance of the solar collector test section.

The proposed mathematical model ($Ho$), at the beginning of its derivation, assumed differing front and rear ambient temperatures. This is an important assumption because only with such an assumption can the model properly predict the temperatures of the collector surfaces. The estimation of the collector rear surface temperature is required because the collector is to be modelled as an integral part of a building; its rear surface temperature is directly related to the heat loss and, thus, the energy consumption of the zone to which the collector is attached.

Since the proposed solar collector is to be structurally-integrated with a building, estimation of the collector surface temperatures is of importance. Therefore, it was decided that comparison between measured and estimated temperatures should also be carried out to confirm the integrity of the mathematically derived model.
From the previous chapter, it is clear that additional heat losses were present in the solar collector test section. These heat losses, not previously accounted for in the mathematical model derivation, consist of conduction heat transfer along the longitudinal axis of the profiled absorber, conduction heat transfer from the extended region of the enclosing edge plates at the side of the collector, and conduction heat transfer from the cross section of the absorber edge. In this chapter, the magnitudes of these heat losses are estimated. They were then taken into account to adjust the estimated thermal performances from the proposed model and the standard (HWB) model.

The thermal performances estimated by the proposed mathematical model (the Ho model) and by a model reported by Duffie and Beckman (1991) (the D&B model) were compared. These figures (before and after accounting for the additional heat losses) as estimated by the models were then used to compare with the figures for $U_\text{L}$ and $F_\text{R}$ as measured in the laboratory. Comparison between the laboratory-measured values of the collector surface temperatures and those estimated by the Ho model with the effect of the multi-layered cover accounted for in the top loss mechanism and by the model reported by Duffie and Beckman for a collector (also with the effect of the multi-layered cover accounted for in the top loss mechanism) (the D&B model) are also presented.

Further validation is then carried out for the condition where front and rear ambient temperatures differ, using measurements obtained from a modified solar simulator. Predictions from both the Ho and HWB models are also compared for the condition when $t_a \neq t_a^\prime$.

4.2. Validation using heat removal factor and overall heat loss coefficient

4.2.1. Estimation of thermal performance using Ho and D&B models

The predicted thermal performance of the proposed solar collector test section in terms of heat collection rate (rate of heat transfer to the air stream) was estimated using the mathematical model derived in this study (Ho) and that reported by Duffie and Beckman (1991) (D&B). Tables 4.1 and 4.2 present the parameters used in the estimations for both models (for meanings
Chapter 4 - Model validation

of terms, refer to the nomenclature at the end of this chapter). These values corresponded to those measured in the laboratory.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\tau_{\alpha})_\varepsilon)</td>
<td>0.68</td>
<td>(d) (m)</td>
<td>0.00696</td>
<td>(W) (m)</td>
<td>1.00</td>
<td>(Pr) (-)</td>
<td>0.7</td>
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<tr>
<td>(k_2) (Wm(^{-1})C(^{-1}))</td>
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<td>(\varepsilon_3) (-)</td>
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<td>(L) (m)</td>
<td>1.00</td>
<td>(C_p) (J kg(^{-1})C(^{-1}))</td>
<td>1005</td>
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<tr>
<td>(d_2) (m)</td>
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<td>(\rho) (kg m(^{-3}))</td>
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<td>(AR) (-)</td>
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<td>0.1198</td>
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Table 4.1: Parameters (properties) used in thermal performance estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Corresponding test number</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_p) (ms(^{-1}))</td>
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</tr>
<tr>
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<td>3.5</td>
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<tr>
<td>1.0</td>
<td>1.0</td>
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<tr>
<td>18.01</td>
<td>18.26</td>
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<td>17.78</td>
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<td>0.134</td>
<td>0.134</td>
</tr>
<tr>
<td>406.7</td>
<td>406.7</td>
</tr>
</tbody>
</table>

Table 4.2: Value used for each variable for estimation of thermal performance of the solar collector test section

<table>
<thead>
<tr>
<th>Variable</th>
<th>Corresponding test number</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_1)</td>
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<tr>
<td>19.00</td>
<td>19.00</td>
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<tr>
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<td>5.47</td>
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</tr>
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<tr>
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<tr>
<td>3.87</td>
<td>4.01</td>
</tr>
<tr>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>(Q_s) (W)</td>
<td>194.53</td>
</tr>
</tbody>
</table>

Table 4.3: Estimated heat transfer coefficients and heat collection rates (before revision for additional heat losses) by Ho and D&B models
The heat collection rates $Q_s$ estimated by the Ho and D&B models are presented in Table 4.3. Note that the additional heat losses have not been taken into account in these estimations. To adjust for the additional heat losses, the model-predicted heat collection rates estimated above in Table 4.3 were reduced by an amount equal to the additional heat losses. These additional heat losses, estimated by second-order differential equations, are presented next in the following section.

4.2.2. Estimation of additional heat loss in the experimental work

In the laboratory test system, routes for additional heat loss from the collector test unit (Figure 4.1) were identified. These heat losses were due to axial conduction along the absorber flow direction (x-direction), lateral conduction through the absorber plate to its edge (y-direction), and conduction from the extended regions of the enclosing edge plates (z-direction). These heat loss mechanisms were not previously taken account of in the mathematical model derivation.

It was reported (Phillips, 1979) that the error in thermal performance prediction may be in the range from 12% to 30% when axial conduction alone is neglected. Therefore, it is essential to include these heat loss mechanisms in the estimation.

Figure 4.1: Additional heat flow directions from the collector
Expressions for these heat loss mechanisms were derived by considering the heat balance for each of the paths identified. Each of these heat balances resulted in a second order differential equation. While the full derivations are presented in Appendix A4, the main equations are shown below:

**Additional conduction heat loss from absorber along longitudinal axis (x-direction)**

\[
Q_{\text{ext}} = -kA_x r_1 \left( T_{\text{tip}} - C_3 \right) - \left( T_h - C_3 \right) \cosh(r_1 L_1) \sinh(r_1 L_1) 
\]  \( \ldots(4.1) \)

**Axial heat loss between absorber and edge plate (y-direction -- normal to air flow direction)**

\[
Q_{\text{profile-side}} = -h_3 y \Delta y 
\]

Figure 4.2: Consideration of parameters for additional heat flow in x-direction

Figure 4.3: Consideration of parameters for additional heat flow in y-direction
\[ Q_{\text{profile-side}} = -kA_x r_4 \left( (T_b + C_6) \sinh(r_3 y) + \frac{(T_{np} + C_6) - (T_e + C_6) \cosh\left(\frac{r_i W}{2}\right)}{\cosh\left(\frac{r_i W}{2}\right)} \cosh(r_3 y) \right) \] 

...(4.2)

**From edge plate-absorber contact to outside**

![Diagram](image)

Figure 4.4: Consideration of parameters for additional heat flow in z-direction

\[ Q_{\text{extend-edge}} = Q_{\text{extend-edge1}} - Q_{\text{extend-edge2}} \] 

...(4.3)

\[ Q_{\text{extend-edge}} = kA_x \frac{dT}{dz}\bigg|_{z=0} - kA_x \frac{dT}{dz}\bigg|_{z=L_3} \]

\[ Q_{\text{extend-edge}} = kA_x r_4 C_{15} - kA_x r_4 \left[ C_{14} \sinh(r_4 L_2) + C_{15} \cosh(r_4 L_2) \right] \] 

...(4.4)

where

\[ T_{e1} = \frac{r_4}{C_{16} C_{18} \sinh\left(\frac{r_4 L_2}{2}\right)} T_e - \frac{r_4}{C_{16} C_{18} \sinh\left(\frac{r_4 L_2}{2}\right)} \left[ 1 - \cosh\left(\frac{r_4 L_2}{2}\right) \right] C_9 \]

\[ + \frac{r_5 r_6}{C_{16} C_{18} \sinh(r_5 L_3) \sinh(r_6 L_4)} t_{np}' \]

\[ - \frac{r_5}{C_{16} C_{18} \sinh(r_5 L_3)} \left\{ \left[ 1 - \cosh(r_5 L_3) \right] + \frac{r_5}{C_{16} \sinh(r_5 L_3)} \left[ 1 - \cosh(r_5 L_1) \right] \right\} t_a \]

\[ + \frac{r_6}{C_{17} \sinh(r_6 L_4)} \left[ 1 - \cosh(r_6 L_4) \right] \] 

...(4.5)
\[ T_{e2} = \frac{r_5}{C_{17} \sinh (r_5 L_3)} T_{e1} + \frac{r_6}{C_{17} \sinh (r_6 L_4)} t_{ip} \]

\[ - \left\{ \frac{r_5}{C_{17} \sinh (r_5 L_3)} \left[ 1 - \cosh (r_5 L_3) \right] + \frac{r_6}{C_{17} \sinh (r_6 L_4)} \left[ 1 - \cosh (r_6 L_4) \right] \right\} t_a \]  \( \ldots (4.6) \)

\[ C_{18} = 1 - \frac{r_5^2}{C_{16} C_{17} \sinh^2 (r_5 L_3)} \]  \( \ldots (4.7) \)

\[ C_{17} = \frac{r_5 \cosh (r_5 L_3)}{\sinh (r_5 L_3)} + \frac{r_6 \cosh (r_6 L_4)}{\sinh (r_6 L_4)} \]  \( \ldots (4.8) \)

\[ C_{16} = \frac{r_5 \cosh \left( \frac{r_4 L_2}{2} \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} + \frac{r_5 \cosh (r_5 L_3)}{\sinh (r_5 L_3)} \]  \( \ldots (4.9) \)

\[ C_{15} = \frac{(T_e - C_9) - (t_{ip}'' - C_9) \cosh \left( \frac{r_4 L_2}{2} \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} \]  \( \ldots (4.10) \)

\[ C_{14} = t_{ip}' - C_9 \]  \( \ldots (4.11) \)

\[ C_{13} = \frac{\cosh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} T_{e2} + \frac{\cosh [r_6 (L_2 + L_3)]}{\sinh (r_6 L_4)} t_{ip}' \]

\[ - \frac{\cosh [r_6 (L_2 + L_3)] - \cosh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} t_a \]  \( \ldots (4.12) \)

\[ C_{12} = \frac{\sinh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} T_{e2} - \frac{\sinh [r_6 (L_2 + L_3)]}{\sinh (r_6 L_4)} t_{ip}' \]

\[ + \frac{\sinh [r_6 (L_2 + L_3)] - \sinh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} t_a \]  \( \ldots (4.13) \)

\[ C_{11} = - \frac{\cosh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} T_{e1} + \frac{\cosh (r_5 L_3)}{\sinh (r_5 L_3)} T_{e2} - \frac{\cosh (r_5 L_2) - \cosh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} t_a \]  \( \ldots (4.14) \)

\[ C_{10} = - \frac{\sinh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} T_{e1} - \frac{\sinh (r_5 L_3)}{\sinh (r_5 L_3)} T_{e2} + \frac{\sinh (r_5 L_2) - \sinh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} t_a \]  \( \ldots (4.15) \)

\[ C_9 = \frac{U_c t_a + h_2' T_\infty}{U_c + h_2'} \]  \( \ldots (4.16) \)
\[ C_8 = -\frac{\cosh(r_4 L_2)}{\sinh\left(\frac{r_4 L_2}{2}\right)} T_e + \frac{\cosh\left(\frac{r_4 L_2}{2}\right)}{\sinh\left(\frac{r_4 L_2}{2}\right)} T_{el} + \frac{\cosh(r_4 L_2) - \cosh\left(\frac{r_4 L_2}{2}\right)}{\sinh\left(\frac{r_4 L_2}{2}\right)} C_9 \quad \ldots (4.17) \]

\[ C_7 = -\frac{\sinh(r_4 L_2)}{\sinh\left(\frac{r_4 L_2}{2}\right)} T_e - T_{el} - \frac{\sinh(r_4 L_2) - \sinh\left(\frac{r_4 L_2}{2}\right)}{\sinh\left(\frac{r_4 L_2}{2}\right)} C_9 \quad \ldots (4.18) \]

\[ C_6 = \frac{I(\tau \alpha)_e}{h_3 + k_2/d_2} - C_3 \quad \ldots (4.19) \]

\[ C_5 = \frac{(T_{wp} + C_6) - (T_b + C_6) \cosh\left(\frac{r_4 W}{2}\right)}{\sinh\left(\frac{r_4 W}{2}\right)} \quad \ldots (4.20) \]

\[ C_4 = T_b + \frac{I(\tau \alpha)_e}{h_3 + k_2/d_2} - C_3 \quad \ldots (4.21) \]

\[ C_3 = \frac{(h_3 T_{wp} + k_2/d_2) T_e}{(h_3 + k_2/d_2)} \quad \ldots (4.22) \]

\[ C_2 = \frac{(T_{wp} - C_1) - (T_b - C_1) \cosh(r_1 L_1)}{\sinh(r_1 L_1)} \quad \ldots (4.23) \]

\[ C_1 = T_b - C_1 \quad \ldots (4.24) \]

\[ r_1 = \sqrt{\frac{W}{kA_x(h_3 + k_2/d_2)}} \quad \ldots (4.25) \]

\[ r_2 = -\sqrt{\frac{W}{kA_x(h_3 + k_2/d_2)}} \quad \ldots (4.26) \]

\[ r_3 = \sqrt{\frac{L}{kA_x(h_3 + k_2/d_2)}} \quad \ldots (4.27) \]
\[ r_4 = \sqrt{\frac{L}{kA_x}}(U_e + h_2') \]  
\[ r_5 = \sqrt{\frac{L(U_f + h_f')}{kA_x}} \]  
\[ r_6 = \sqrt{\frac{2h_f'L}{kA_x}} \]

To estimate the heat loss due to each path, appropriate variables such as air temperatures and irradiances were substituted into Equations 4.1 to 4.30. Table 4.4 presents the variables used for each corresponding test.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corresponding test number</th>
<th>Corresponding test number</th>
<th>Corresponding test number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beginning</td>
<td>Final</td>
<td>Beginning</td>
</tr>
<tr>
<td>( t_i ) (°C)</td>
<td>17.78</td>
<td>35.06</td>
<td>17.40</td>
</tr>
<tr>
<td>( d_2 ) (m)</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>( h_2 ) (Wm(^{2}°C^{-1}))</td>
<td>4.82</td>
<td>4.74</td>
<td>6.76</td>
</tr>
<tr>
<td>( h_3 ) (Wm(^{2}°C^{-1}))</td>
<td>5.63</td>
<td>5.53</td>
<td>7.89</td>
</tr>
<tr>
<td>( k_2 ) (Wm(^{2}°C^{-1}))</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>( t_a ) (°C)</td>
<td>18.01</td>
<td>18.26</td>
<td>17.79</td>
</tr>
<tr>
<td>( t_e ) (°C)</td>
<td>22.02</td>
<td>27.97</td>
<td>20.78</td>
</tr>
<tr>
<td>( t_{ow} ) (°C)</td>
<td>28.48</td>
<td>41.09</td>
<td>25.91</td>
</tr>
<tr>
<td>( t_{up} ) (°C)</td>
<td>18.01</td>
<td>18.26</td>
<td>17.79</td>
</tr>
<tr>
<td>( t_{up}'' ) (°C)</td>
<td>21.00</td>
<td>21.00</td>
<td>21.00</td>
</tr>
<tr>
<td>( t_{up}''' ) (°C)</td>
<td>21.00</td>
<td>21.00</td>
<td>21.00</td>
</tr>
<tr>
<td>( t_{el} ) (°C)</td>
<td>17.78</td>
<td>35.03</td>
<td>17.40</td>
</tr>
<tr>
<td>( t_{e2} ) (°C)</td>
<td>19.23</td>
<td>35.97</td>
<td>18.89</td>
</tr>
<tr>
<td>( T_1 ) (°C)</td>
<td>19.65</td>
<td>20.72</td>
<td>19.11</td>
</tr>
<tr>
<td>( T_{el} ) (°C)</td>
<td>34.86</td>
<td>48.34</td>
<td>31.37</td>
</tr>
<tr>
<td>( T_{e2} ) (°C)</td>
<td>44.26</td>
<td>55.88</td>
<td>38.17</td>
</tr>
<tr>
<td>( U_e )</td>
<td>6.25</td>
<td>6.25</td>
<td>6.25</td>
</tr>
<tr>
<td>( W ) (Wm(^{2}°C^{-1}))</td>
<td>1.57</td>
<td>1.57</td>
<td>1.57</td>
</tr>
<tr>
<td>( I_p ) (ms(^{-1}))</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4.4: Parameters used for estimation of total extra (additional) heat transfer (axial conduction, heat loss from extended region of the enclosing edge plates and that from edge of absorber) (see nomenclature for meaning of variables and their units)
Chapter 4 - Model validation

<table>
<thead>
<tr>
<th>Heat loss (W)</th>
<th>Corresponding test number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Beginning</td>
</tr>
<tr>
<td>$Q_{\text{in}}$ (inlet)</td>
<td>14.31</td>
</tr>
<tr>
<td>$Q_{\text{out}}$ (outlet)</td>
<td>21.30</td>
</tr>
<tr>
<td>$Q_{\text{extend-edge}}$</td>
<td>2.39</td>
</tr>
<tr>
<td>$Q_{\text{profile-side}}$</td>
<td>3.04</td>
</tr>
<tr>
<td>Total estimated</td>
<td>41.04</td>
</tr>
<tr>
<td>additional heat loss rate (W)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Estimated additional heat loss from test section (see nomenclature for meaning of variables and their units).

Note: $Q_{\text{extend-edge}} = Q_{\text{extend-edge1}} - Q_{\text{extend-edge2}}$

To revise the predicted $U_L$ and $F_R$ values while accounting for these additional heat losses, their magnitudes (estimated using Equations 4.1 to 4.30 and presented in Table 4.5) were subtracted from the heat transferred to the air stream as estimated by the models (and presented in Table 4.3). Note that the figures used to revise the estimated $U_L$ and $F_R$ values correspond to the beginning and the final readings in each laboratory measurement presented in Chapter 3 (Figure 4.5).

![Figure 4.5: Typical collector characteristic generated by regression fitted line using laboratory measurement](image)

For the three collector operating conditions tested, Table 4.6 presents a comparison between the experimentally measured values for $F_R$ and $U_L$ and
those predicted by the model derived in Chapter 2 (Ho model) and by the Duffie and Beckman model (after additional heat loss is accounted for).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Measured values</th>
<th>Predicted values after additional heat loss is accounted for:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ho</td>
</tr>
<tr>
<td></td>
<td>$U_L$</td>
<td>$F_R$</td>
</tr>
<tr>
<td>1</td>
<td>12.15</td>
<td>0.78</td>
</tr>
<tr>
<td>2</td>
<td>10.71</td>
<td>0.82</td>
</tr>
<tr>
<td>3</td>
<td>12.64</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Table 4.6: Comparison of measured $U_L$ (Wm$^{-2}$°C$^{-1}$) and $F_R$ (-) values with prediction (after additional heat loss is accounted for), based on models of Ho and of Duffie and Beckman (1991)

Inspection of Table 4.6 shows that both the Ho model and the Duffie and Beckman (D&B) model predict the same values for $U_L$ and $F_R$. This is so because under the condition of equal front and rear ambient temperatures, the Ho model is is equivalent to (and reduces to) the D&B model. This agrees with the findings in Chapter 2 (see section 2.4). Both the Ho and D&B models predict a value for $U_L$ to within 28% (on average) of that measured. The discrepancy between measured and predicted values is considered to be caused by random experimental uncertainty in the measurements.

### 4.3. Validation using surface temperatures

Surface temperatures were used for carrying out further validation. This was done by plotting measured values (from the laboratory work) against the corresponding predicted values (from the mathematical models). The closer the gradient and the correlation coefficient ($r$) of the fitted regression line are to unity, then the better and more consistently can the model predict the surface temperatures.
Figure 4.6 presents the relationship between predicted and measured temperatures on outer and inner cover surfaces and absorber surface. Inspection shows that, when plotted against the measured surface temperatures, values predicted by the Ho and D&B models gave a regression fitted line with gradient and correlation coefficient very close to unity, indicating that both models, under the condition of equal front and rear ambient temperatures, predict equally well the surface temperatures, these being in agreement with the measured values.

4.4. Predictions of thermal performance for differing front and rear ambient temperatures

The preceding condition in which the thermal performance of the collector test section was measured was that of equal front and rear ambient temperatures. Although the proposed model was considered to be validated (to within the experimental uncertainty) for that particular condition, it is essential to validate its performance for the condition when front \( t_a \) and rear \( t_a' \) ambient temperatures differ, i.e. \( t_a \neq t_a' \), and to compare the findings with the standard Hottel-Whillier-Bliss (HWB) analysis. It is also important to determine the conditions of front / rear ambient temperature difference and of levels of backing insulation that give rise to any significant differences between predictions made using the proposed new model and those using the existing HWB approach. These issues are addressed in this section.
The concept of the equivalent ambient temperature \( t_a \), introduced in Chapter 2, enables the thermal performance of a collector to be estimated for different front and rear ambient temperature in terms of the well-known parameters \( U_L \) and \( F_R \). By contrast, the standard HWB treatment is unable to account for such a situation, the model strictly being applicable to the condition when front and rear ambient temperatures are equal. When faced with the situation of differing ambient temperatures, in the past researchers frequently addressed the problem by simply averaging the front and rear ambient temperatures. However, Ho and Loveday (1997) suggested that the effect of differing front and rear ambient temperatures could be better accounted for by replacing the ambient temperature \( t_a \) in a standard HWB analysis with a new ambient temperature \( t_a'' \) weighted in terms of the front and rear loss coefficients and defined as:

\[
  t_a'' = \frac{U_i' t_a' + U_b t_a'}{U_i' + U_b} \quad \ldots(4.31)
\]

It can thus be seen that the use of \( t_a'' \) is an improvement upon simply averaging the front and rear ambient temperatures, but that it is not as accurate (or rigorous) as using \( t_a' \) as derived in this thesis. Clearly, the magnitude of any discrepancies between the approaches is dependent not only upon the front / rear ambient temperature difference, but also upon the conductance of any rear insulation; all approaches should give identical results when \( t_a = t_a' \) as shown in sections 4.2 and 4.3 (other effects, such as top cover heat loss, being the same).

In order to investigate the above effects, comparison is made between values for \( U_L \) predicted using the Ho model (that is, using \( t_a' \)) and using the standard HWB analysis employing \( t_a'' \). Figure 4.7a shows predicted values for \( U_L \) as a function of \((t_a' - t_a')\) using the two approaches, for a fixed rear conductance \((k_2 / d_2)\) of 0.4 Wm\(^{-2}\)C\(^{-1}\) (a typical value for profiled steel with insulated backing), and fixed front ambient temperatures of 1 °C and 20 °C. It can be seen that when \((t_a' - t_a')\) is zero (that is, \( t_a = t_a' \)), both approaches (Ho and HWB) give identical results, as expected. As the front / rear ambient temperature difference increases, predictions by the Ho and HWB models progressively diverge, though the discrepancy between the predictions is small for the chosen level of backing insulation. In addition, the value for \( U_L \)
is seen to be affected by the inlet air temperature to the collector (equal to the front ambient temperature of 1 °C or 20 °C).

Figure 4.7b shows a similar set of relationships to Figure 4.7a, but for a much greater rear conductance \((k_2 / d_2)\) of 50000 Wm\(^{-2}\)°C\(^{-1}\); in other words, this corresponds to a collector where the rear insulation has effectively been replaced by a thin metal sheet (for example, the thickness of a metal absorber plate only). Again, at \((t_a' - t_a)\) equal to zero (that is, \(t_a = t_a'\)), both Ho and HWB models give identical predictions for \(U_L\). The values for \(U_L\) in Figure 4.7b are greater than those in figure 4.7a because of the reduction in insulation. As \((t_a' - t_a)\) increases, predictions for \(U_L\) from both models progressively diverge, but there is now the appearance of some minimum 'turning points'. Since the rear heat loss coefficient \(U_b\) is now chiefly a function of the convective and radiative heat transfer, and that the rear convection coefficient \(h_4\) is a function of the temperature difference between the rear surface of the collector and the rear ambient environment \((T_4 - t_a')\), it is thought that these turning points are related to changes in direction of heat flow through the rear of the collector. However, caution is required here; once the condition is reached where \(t_a' > T_4\), it may become necessary to calculate a value for the rear convection coefficient \(h_4\) using a different correlation (warm fluid to a cooled plate). This could affect the predictions of both approaches beyond a certain front / rear ambient temperature difference, and it is recommended that this be further investigated. What can be concluded is that for very little or no rear insulation, and for front / rear ambient temperature differences beyond, say, 15 °C, significant errors can arise if collector performance is predicted using the standard HWB analysis, even with the adoption of a weighted ambient temperature \(t_a'\). It is then necessary to utilise the model presented in this thesis.
Figure 4.7a: Overall heat transfer coefficient versus rear / front ambient temperature difference for $t_r = 1^\circ C$ and $k_2 / d_2 = 0.4 \text{ Wm}^{-2} \text{ C}^{-1}$

Figure 4.7b: Overall heat transfer coefficient versus rear / front ambient temperature difference for $t_r = 1^\circ C$ and $k_2 / d_2 = 50000 \text{ Wm}^{-2} \text{ C}^{-1}$
Figure 4.8a: Overall heat transfer coefficient versus rear thermal conductance for \( t_i = 1^\circ C \)

Figure 4.8b: Overall heat transfer coefficient versus rear thermal conductance for \( t_i = 20^\circ C \)

Figure 4.8a and 4.8b show values for \( U_L \) as predicted using Ho and HWB models as a function of rear conductance \( (k_y/d_z) \) for two collector inlet temperatures, 1 °C and 20 °C, respectively. Note that the front ambient temperature \( t_a \) is fixed at 1 °C throughout, and the rear ambient temperatures are fixed at either 45 °C or 17 °C (plots - not shown - for the case of \( t_a = t_a' \) gave identical values for both models, as would be expected).

Inspection shows that \( U_L \) increases as rear conductance increases; in addition, the predictions in \( U_L \) from both models begin to diverge at a rear
conductance value of about 2 Wm\(^{-2}\)°C\(^{-1}\) (that is, a thermal resistance of 0.5 m\(^2\)°CW\(^{-1}\)). For conductances less than this (say 0 ~ 2 Wm\(^{-2}\)°C\(^{-1}\)) predictions from either modelling approach give almost the same results. At very large conductances (> 10 Wm\(^{-2}\)°C\(^{-1}\), say), the differences in \(U_L\) as predicted by the models tend to become fairly constant. These findings are for the (typical) front / rear ambient temperature differences used.

It can thus be concluded that when the rear conductance exceeds about 2 Wm\(^{-2}\)°C\(^{-1}\), and the front / rear ambient temperature difference exceeds, say 15 °C, prediction of the thermal performance of a structurally-integrated air-heating solar collector using the standard HWB method could result in error. For these situations, predictions are likely to be more accurate if the modelling approach presented in this thesis (that is, the \(t_a^*\) concept) is adopted. These findings are particularly relevant to the cases of multi-functional photovoltaic facades, and / or situations of air temperature stratification in heated indoor spaces.

As regards the application being treated in this research, that of a profiled steel collector with backing insulation, these is no significant difference between modelling based upon \(t_a^*\) (the Ho model) or upon \(t_a''\) (the HWB model), since the rear conductance is about 1.3 Wm\(^{-2}\)°C\(^{-1}\)

4.5. Validation for condition of differing front and rear ambient temperatures

Thus far, the validation of the proposed mathematical model has been carried out (sections 4.2 and 4.3) for the condition of equal front and rear ambient temperatures (\(t_a = t_a^*\)). However, under such conditions, the proposed model is only partially validated, since it was derived in order to treat situations where front and rear ambient temperatures differ (that is, \(t_a \neq t_a^*\)).

In the previous section (section 4.4), the limits of front / rear temperature difference and of rear insulation level for which the standard Hottel-Whillier-Bliss analysis can be considered inaccurate, have been estimated. It is clear, from the findings in the previous section, that the use of the proposed mathematical model becomes relevant when the rear conductance exceeds about 2 Wm\(^{-2}\)°C\(^{-1}\) and the rear-front ambient temperature difference exceeds about 15 °C. These figures of 'critical' front / rear temperature difference and
rear insulation level were used for determining appropriate values for the rear insulation level and the front / rear ambient temperature difference in the modified solar simulator as described in Chapter 3 (section 3.6.1). Using the measurements presented in section 3.6.1, a more general validation of the model proposed in Chapter 2 can be carried out, and is presented here.

4.5.1. Modified heat transfer correlations

The modified solar simulator has a different geometry and flow regime from that of the original solar simulator. Therefore, a modified set of correlations was used for prediction of the thermal performance when \( t_a \neq t_a'. \)

The air mass flow rate through the collector was 0.07 kgs\(^{-1}\). The corresponding Reynolds number for this flow rate was approximately 6000, suggesting that the flow regime was in the transition region. To estimate the Nusselt number, correlations for both laminar and turbulent flow were used.

For laminar flow, the Nusselt number is given by (Duffie and Beckman, 1991):

\[
Nu = 5.385 \quad \ldots(4.32)
\]

and for turbulent flow, the correlation for estimation of the Nusselt number is (Duffie and Beckman, 1991):

\[
Nu = 0.0158 \text{Re}^{0.8} \quad \ldots(4.33)
\]

The estimation of Nusselt number for transitional flow was then found by interpolation between Reynolds numbers for laminar and fully turbulent flows (2300 and 10000, respectively). The corresponding convection coefficients were then used in the prediction of the thermal performance of the collector test section (when \( t_a \neq t_a' \)).

The modifications to the collector test section create additional heat losses due to axial conduction and edge 'fin-effect' which must be added to the useful heat output \( Q_u \) predicted by Ho and HWB models. This axial conduction is the heat loss from the collector measurement section by conduction through the absorber plate out of the considered system boundary along and against the flow direction; the 'fin-effect' is the effect of the edge-
plates acting as a heat exchanger extended 'fin' surface, thus contributing to additional heat loss.

The way in which the axial conduction was re-estimated was again by heat balance on the absorber plate in the flow direction and in the direction normal (perpendicular) to the channel air flow direction. The result was a partial differential equation, which was solved using the finite difference approach to yield:

\[
T_{m,n} = \frac{T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1}}{C_1 \Delta x} + C_2 \Delta x + 4
\]

where

\[
C_1 = \frac{1}{ku} \{ h_{r23} + h_1 + h_3 + h_{r4} \} \quad \cdots (4.35)
\]

\[
C_2 = \frac{1}{ku} \{ T_f h_{r23} + t_f h_3 + t_\alpha (h_4 + h_{r4}) + I_T \tau \alpha \} \quad \cdots (4.36)
\]

\[
\Delta x \quad = \quad \text{grid size of elements (m)}
\]

The additional heat loss from the absorber to the ambient environment can then be estimated by applying the following to each boundary of the (1 m²) solar simulator test section:

\[
Q_{\text{additional-absorber}} = ku \Delta T \quad \cdots (4.37)
\]

where

\[
k \quad = \quad \text{conductivity of the absorber (Wm}^{-2}\text{C}^{-1})
\]

\[
u \quad = \quad \text{thickness of the absorber (m)}
\]

\[
\Delta T \quad = \quad \text{temperature difference between adjacent element (°C)}
\]

The results of the model predictions are presented in the following section in terms of the predicted useful energy collection rate, \(Q_u\), corrected for the additional heat loss. Validation is carried out by comparison between the predicted and the measured values of \(Q_u\) across the range of front / rear ambient temperature differences.

4.5.2. Validation
Traditionally, validation is carried out by comparison between the predicted and measured values for $U_L$ and $F_R$. However, for the situation where front and rear ambient temperatures differ, determining values for $U_L$ and $F_R$ from experiment presents difficulties because the values obtained are dependent upon the choice of surrounding ambient temperature ($t_{a,m}$, $t_{a}''$ or $t_{a}^*$) (see section 3.6.1). Therefore, the use of these indices for comparison is not suitable here.

Instead, comparison between predicted and measured values for the useful energy collection rate ($Q_u$) was considered a better approach for validation. This is because of the manner in which $t_{a}^*$ is defined, that is, the equivalent ambient temperature that gives the same useful energy collection rate as occurs with the actual front and rear ambient temperatures ($t_{a}$ and $t_{a}''$, respectively).

Figure 4.9a: Useful heat collection rate versus rear / front temperature difference for $t_i=18^\circ$C and $k_z / d_z = 50000$ Wm$^{-2}$°C$^{-1}$
Figures 4.9a and b show the comparison between measured values for \( Q_u \) versus \( (t_a'-t_a) \), and those predicted using the Ho model (based on \( t_a' \)) and the HWB models (based on \( t_{a,om} \) as defined in Equation 4.31). It can be seen that the useful energy collection rate as predicted by both Ho and HWB models are in good agreement with measured values. However, it is clear that the Ho predictions are consistently closer to the measured values than are the predictions by the HWB model.

It is also noted from Figures 4.9a and b that as \( t_a'-t_a \) increases, the value for \( Q_u \) increases, as heat is 'gained' from the rear environment. Furthermore, the divergence between the Ho prediction and the HWB prediction increases, which is in general agreement with the findings in section 4.4. Note that at the higher values of \( (t_a'-t_a) \), the Ho model gives the better agreement to measured results.
Figure 4.10: Comparison between the predicted and measured useful heat output

However, to quantify more clearly the differences in predictions from Ho and HWB models, Figure 4.10 presents a plot of predicted useful energy collection rates by the solar collector (for both HWB and Ho analysis) versus the measured useful energy collection rate. It can be seen from the comparison of regression-fitted lines for the model predictions with the ideal regression that the Ho model consistently predicts better than the HWB model, especially at the higher values of $Q_u$ (corresponding to heat gain from the rear environment as $t_a$ increases). The model presented, together with the newly-introduced $t_{a^*}$ concept, is therefore considered to have been validated and is shown to predict better than the standard HWB approach when front / rear ambient temperature differences exist and are significant.

4.6. Summary

The proposed mathematical model for predicting the thermal performance of a profiled steel air heating solar collector covered with a multi-layer polycarbonate cover system, as presented in Chapter 2, is considered to have been validated for its proposed application (to within experimental uncertainty). The model was validated under the condition when $t_a = t_a'$. Note that, under this condition, the proposed model is equivalent to that of a standard Hottel-Whillier-Bliss analysis.

The model, which represents an extension to the existing Hottel-Whillier-Bliss analysis, was then used to predict for conditions when $t_a \neq t_a'$, and to predict
over a range of rear insulation levels. It was found that for the conditions when the rear conductance exceeds about 2 Wm\(^{-2}\)°C\(^{-1}\), and the front / rear ambient temperature difference exceeds about 15 °C, predictions of collector performance using the model proposed begin to differ significantly from those obtained using the standard HWB analysis. Note that use of the latter required the introduction of a weighted ambient temperature \(t'_a\). Further validation was then carried out based on measurements made for the condition of differing front and rear ambient temperatures. With the additional extraneous heat losses estimated and accounted for in the predictions, the model presented in this thesis (the Ho model) has been shown to more accurately predict the useful energy collection rate, \(Q_u\), compared with the standard HWB approach. The predictions tend to diverge as the front / rear ambient temperature difference increases, with the Ho model consistently giving the more accurate prediction, as would be expected. It is therefore concluded that the model presented in this thesis is suitable for the prediction of solar collector performance (of the geometry modelled) when front and rear environments are at differing temperatures, and that the introduction of the equivalent ambient temperature concept is a useful approach in this respect.

For the application being considered in this thesis, namely that of a structurally-integrated profiled steel collector with backing insulation (conductance 1.3 Wm\(^{-2}\)°C\(^{-1}\)), the derived model is suitable, with predictions differing very little from those of a standard Hottel-Whillier-Bliss analysis.

For generality, however the proposed mathematical model (the Ho model) is used for thermal performance prediction of such a structurally-integrated air heating solar collector. Chapter 6 presents the application of the proposed mathematical model to evaluate the thermal performance of a typical sports centre building equipped with a facade-integrated air-heating solar collector. This involves use of the derived model as a component of a building thermal simulation program.
Nomenclature

\[ A_e = \text{area of edge plate (1-side) (m}^2) \]
\[ A_s = \text{cross sectional area of absorber (m)} \]
\[ AR = \text{aspect ratio (-)} \]
\[ = \text{(length of plates) / (spacing between plates)} \]
\[ C_p = \text{specific heat capacity of channel fluid (JKg}^{-1}\text{o C}^{-1}) \]
\[ d = \text{spacing between polycarbonate sheet within the cover system (m)} \]
\[ d_2 = \text{thickness of rear insulation (m)} \]
\[ d_e = \text{thickness of edge plate insulation (m)} \]
\[ h_1 = \text{convection heat transfer coefficient between cover outer surface and front ambient environment (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_2 = \text{convection heat transfer coefficient between edge plate and collector channel flow (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_3 = \text{convection heat transfer coefficient between fluid flow and absorber (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_4 = \text{convection heat transfer coefficient between rear surface to rear ambient environment (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{v1u2} = \text{natural convection heat transfer coefficient between the top and middle layers of the cover system (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{v3u2} = \text{natural convection heat transfer coefficient between the middle and bottom layers of the cover system (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_f = \text{heat transfer coefficient between exposed part of the extended region of the edge plate (Wm}^{-2}\text{o C}^{-1}) \]
\[ h' = \text{combined convection and radiation heat transfer coefficient (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{r23} = \text{radiation heat transfer coefficient between absorber and cover surface that forms part of the flow channel (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{r1} = \text{radiation heat transfer coefficient between cover outer surface and sky (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{r1u2} = \text{radiation heat transfer coefficient between the top and middle layers of the cover system (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{r3u2} = \text{radiation heat transfer coefficient between the middle and bottom layers of the cover system (Wm}^{-2}\text{o C}^{-1}) \]
\[ h_{r4} = \text{radiation heat transfer coefficient between rear surface and rear environment (Wm}^{-2}\text{o C}^{-1}) \]
\[ I(\tau \alpha)_e = \text{solar irradiance falls on absorber (profiled metal) (Wm}^{-2}) \]
\[ k_e = \text{conductivity of rear insulation (Wm}^{-1}\text{o C}^{-1}) \]
\[ k_e = \text{conductivity of edge plate insulation (Wm}^{-1}\text{o C}^{-1}) \]
\[ L = \text{length of collector test section (m)} \]
$L_1$ = vertical distance from collector system boundary (inlet or outlet of test section) to end of profile cladding (m)

$L_2$ = spacing between absorber trough to cover inner surface (Figure 4.4) (m)

$L_3$ = width of the metal frame that support the cover system (Figure 4.4) (m)

$L_4$ = depth of the edge plate completely exposed to front ambient environment (m)

$m$ = mass flow rate of channel fluid (kgs$^{-1}$)

$MRT$ = mean radiant temperature (°C)

$P_w$ = wetted perimeter of the air channel (m)

$Pr$ = Prandtl number (-)

$Q_{ext}$ = heat loss along the longitudinal axis from the absorber (W)

$Q_{ext-edg}$ = heat loss from collector test section by conduction from extended region of the edge plate (W)

$Q_{profile-side}$ = heat loss from absorber to collector edge by conduction (W)

$Q_u$ = heat collection estimation before adjustment for extra (additional) heat losses (W)

$t_a$ = front ambient temperature (°C)

$t_{eq}$ = equivalent ambient temperature defined by Ho and Loveday (1997) (°C)

$t_a'$ = equivalent ambient temperature defined in Chapter 2 (°C)

$t_e$ = edge plate temperature (°C)

$t_{env}$ = environment temperature inside air channel (°C)

$t_f$ = fluid temperature (°C)

$t_i$ = inlet temperature (°C)

$t_{ip}$ = temperature of the absorber at either end (length-wise) and edge (width-wise) of profiled metal (°C)

$t_{ip'}$ = temperature at the tip of the exposed region of the edge plate (°C)

$t_{ip''}$ = surface temperature at which edge plate intercept absorber (°C)

$T_4$ = temperature of the rear surface of the collector (°C)

$T_b$ = temperature of the absorber at inlet or outlet of the collector test section (°C)

$T_c$ = surface temperature at the middle of channel portion of edge plate (°C)

$T_{c1}$ = surface temperature of edge plate where the cover system meet the edge plate (°C)

$T_{c2}$ = temperature of edge plate where metal frame that support the cover system meets that of the exposed region (m)

$T_s$ = temperature of the absorber at the element considered (for expression derivation). See Figure 4.2 (°C)

$T_{s'}$ = temperature of the absorber at the element considered (for expression derivation). See Figure 4.3 (°C)
Chapter 4 - Model validation

\( T_o \) = from ambient air temperature to the sheltered region of the absorber \([= t_1 (T_{w1}) \text{ for inlet region, } = t_o (T_{w2}) \text{ for outlet region}] \) (°C)

\( U_f \) = metal frame's U-value \((\text{Wm}^{-2}\text{oC}^{-1})\)

\( U_e \) = heat transfer coefficient for edge plate \((\text{Wm}^{-2}\text{oC}^{-1})\)

\( U_t \) = front heat transfer coefficient from rear cover surface to front ambient environment \((\text{Wm}^{-2}\text{oC}^{-1})\)

\( U_r \) = front heat transfer coefficient from rear cover surface to front ambient environment (also account for edge loss) \((\text{Wm}^{-2}\text{oC}^{-1})\)

\( V_r \) = cover surface or meteorological wind speed \((\text{ms}^{-1})\)

\( x \) = distance from collector system boundary (at inlet or outlet of test section) to element considered along the absorber in the flow direction (for expression derivation (see Figure 4.2) (m)

\( y \) = distance from centre line of the collector test section to element considered along the absorber normal to the flow direction (for expression derivation (see Figure 4.3) (m)

\( z \) = distance from absorber of the collector test section to element considered alone the edge plate (for expression derivation (see Figure 4.4) (m)

\( \varepsilon_r \) = emissivity of cover layers (-)

\( \varepsilon_3 \) = emissivity of absorber surface [fluid channel side] (-)

\( \varepsilon_4 \) = rear surface emissivity (-)

\( \rho \) = density of air in the solar collector \((\text{kgm}^{-3})\)
References


Chapter 5 - Case study: Use of the proposed solar collector system in a sports centre

5.1. Introduction

In the previous chapters, a mathematical model was presented which describes the thermal performance of the profiled metal solar air heating collector. The model can account for the situation when ambient temperatures differ at the front and rear of a collector; the model has been shown to be suitable for analysing the performance of building-integrated solar collector system. The next stage of the work was to evaluate, using thermal simulation, the performance of such a building-integrated collector system in the UK climate, with reference to an appropriate case study building. The evaluation includes estimation of cost-effectiveness, and so performance over a sufficiently long period of time, such as a year, is required. This, together with a tool which designers can use for conducting such evaluations, are described in the remaining chapters. In this chapter, the selection of a suitable case study building is described.

5.1.1. Selection of building type

In order to assess the performance and the cost-effectiveness of the proposed solar collector system, it is necessary to determine which types of buildings are more likely to adopt such a solar heating technique. Although the proposed air heating solar collector, if proved effective, can be applied to any building with profiled metal cladding as an outer skin, at present there may be certain building types that would more readily use the system. Thus buildings that require a large supply of outdoor fresh air could benefit. These might include auditoria (large numbers of occupants) or buildings that house certain industrial processes (SEIA, 1995). The attention currently being paid to displacement ventilation systems (usually requiring a 100% fresh air supply leading to improved indoor air quality) is likely to lead to continuing and increasing use of such ventilation; the air collector could be directly integrated with such systems. Many existing buildings already utilise profiled metal cladding, particularly retail park outlets.

A study (Haughey, 1990) has been conducted to assess the 'do-it-yourself' (DIY) outlets in terms of the potential for increasing energy efficiency.
Although DIY outlets-type buildings with profiled metal cladding are appropriate for integration of the proposed solar collector, it may not at present be welcomed because of the trend by this sector to keep accommodation costs as low as possible (Gibb, 1996). Supermarket chains are interested in adopting energy conservation measures into their properties (Gibb, 1996), but because such buildings use both heating and cooling (HVAC) throughout the year, the energy saving would be comparatively small compared to buildings which are only heated but not cooled. Offices with profiled-clad facades can also be found in UK, but because offices are frequently heated and cooled (HVAC) the same reason as for supermarket chains might restrict the use of the solar air-heating system.

To date, the leisure sector has demonstrated a significant interest in energy efficiency. This can be reflected by the number of publications about energy conservation (Sports Council, 1985, 1986, 1987a,b,c, 1988a) in sports centres. In view of this, it was decided to investigate sports-dedicated buildings with a view to using them as a possible case study in this research. In addition, they may be regarded as a test of the performance of the technology, falling somewhere between buildings requiring high rates of fresh air supply, and low-cost retail outlets requiring merely secure cover for storage / sale of products.

5.2. Sports / leisure buildings

There was a major increase in indoor dry sports facility development between about 1966 to 1981. These developments encompass a wide range of building forms catering for a diverse number of activities. The main effort has been on providing buildings for participation in sports, combined with a variety of spectator facilities; such sports include badminton, squash, basketball, martial arts; some other sports, such as 5-a-side soccer and indoor hockey, have developed within the new building type (Sports Council, 1981).

The number of people in the UK actively participating in indoor sports is on the increase. It has been predicted (Leisure Consultants, 1996) that the increase will continue to be so at least to the year 2000. Following a period of growth, the number of sports halls being built or proposed is now decreasing (Sports council, 1991a, 1992, 1993). For example, the number of new 'dry' sports hall constructions completed during the years 1991, 1992 and 1993
were 53, 49 and 24, respectively (Sports Council, 1991b, 1992, 1993). A further 14 new sports halls were under construction in 1993 (Sports Council, 1993). The trend has been to use profiled metal as roofing/walling materials of sports halls. Since the early 1980s, the use of profiled steel cladding in sports centres as roofing and/or walling materials has become popular. Individual designs (Sports Council, 1980 and 1989) and recommendations for sports centre constructions (Sports Council, 1986 and 1990; Scottish Sports Council, 1992) have shown such a trend. There is thus a significant number of such facilities that are available for possible retrofitting of the collector system, together with a number of new-build applications.

The net expenditure of all local authorities in England and Wales on indoor sports facilities constitutes 30.3% of the total budget for leisure and recreation facilities (CIPFA, 1995).

Energy costs are second only to staffing costs as the largest expenditure item facing sports centre managers. In 'dry' sports centres - sports centres without swimming pools - energy costs accounted for a sixth of all running costs in 1988 (Sports Council, 1988a). Accordingly, the need for energy efficiency has been treated as a priority by the sector (Sports Council, 1985, 1987a, b and c). Space heating constitutes about 75% of the total energy use in sports centres without a swimming pool (DOE, 1992). Recently, energy efficiency has again been stressed as being of importance during the design process of sports halls (Harper, 1994), in common with many other types of building. This clearly shows the commitment of the sector to energy conservation.

The development of the Standardised Approach to Sports Hall (SASH) centres has been promoted and encouraged in England and Wales by the Sports Council (Sports Council, 1988b). First introduced in 1993, one aim of the design of SASH centres was to obtain the best possible value for money; therefore, they have incorporated the most up-to-date energy conserving and environmental features (Sports Council, 1988a). Until 1988, there were 27 SASH centres that were under development in England and Wales (Sports Council, 1988b). One of the options for the construction of the roof / wall was to use profiled metal. Despite the good energy performance of SASH buildings in general, it was considered (Sports Council, 1988b) that there was still some scope for improvement. Modification of the SASH designs for future projects might be necessary to achieve this (Sports Council, 1988b). This
could include the use of building facades to utilize solar energy, should the proposed system be proven cost-effective.

In 1987, the development of the Low Cost Hall (LCH) was investigated (Sports Council, 1991a). This consists of a lightweight industrial type construction normally comprised of insulated metal cladding. Its purpose was as a quick and cheap addition to sites to complement existing facilities and services. The reason for this development was that restrictions on capital expenditure forced Local Authorities to look at new ways of providing for sport whilst the supply of new centres fell short of demand. One of the performance specification requirements is that available methods for upgrading the structure (either in terms of the addition of functional areas, or the improvement of specification such as the addition of insulation and heating) should be demonstrated and extra over costs indicated. The solar collector system considered in this research would provide an improvement in insulation and heating, making it a potential retrofit option for energy saving, and satisfying the performance specification requirement stated above (Sports Council, 1990, 1991a).

The use of solar collectors for sports premises to aid energy efficiency installation was tried in the early eighties (Association of Local Authorities of Northern Ireland, 1981). More recently, a project consisting of a sports hall (with a swimming pool) having a photovoltaic array as cladding has been constructed. An 80 year payback period was estimated for the installation (Gibson, 1996). This shows the willingness of the sector to invest in energy efficient features, despite the potentially long payback.

Many sports hall buildings now use radiant heating in the sports hall itself; however, about half of the existing sports halls use boiler-heated ducted warm air heating systems (Gibson, 1996). The latter makes the integration of the proposed air heating solar collector particularly suitable. However, it is also possible to integrate the proposed air heating solar collector for the preheating of ventilation air to a radiantly-heated sports hall.

In summary, the foregoing information indicates that the sports / leisure sector would be prepared to consider the installation of energy-saving technology within its facilities. In addition, the extensive use of profiled metal cladding for sports halls together with the fact that a significant fraction of
such buildings employ warm air heating, suggests that the air-heating solar collector proposed in this research would be a suitable feature to incorporate in a sports hall design. It was decided to consider the case of a retrofit collector installation to a boiler-heated ducted warm air system; this would offer a realistic and 'conservative' trial of the technology particularly in a retrofit context, and hence this was chosen as the case study for further investigation. It is necessary to produce a 'standard' typical sports hall design in terms of the following information:

1. Building dimensions, layout, and construction
2. heating / cooling requirements
3. method of heating used (existing systems)
4. method of ventilation used (existing systems)
5. building use pattern (occupancy)
6. lighting and other gains

The development of the 'standard' sports hall to be used in the thermal simulation exercise is described next.

5.3. The 'standard' case study building

5.3.1. Dimensions and layout

The earlier sports halls were dimensionally based on the game of tennis, which had proved to be in little demand (presumably due to the uneconomic use of the whole space by so few people). There was thus a need for a much more flexible approach and range of hall sizes to meet differing needs.

The trend between 1971 and 1981 was to group facilities together under one roof, thereby offering a number of obvious advantages. Management and staffing costs represent a significant part of the annual budget for running sports facilities, and grouping them together gives savings; bars and social facilities can combine; families, groups and individuals can have a choice of activities at the same centres and visitors become aware of other activities; this can be achieved with the same staff team (Sports Council, 1981). Most sports buildings (with no swimming pool) are arranged in such a way that the main activity area (the sports hall) is on one side of the building; on the other side of the building are rooms such as offices and changing rooms, making
the plan of the whole building approximately square (Sports Council, 1981). In halls categorised as 'small', the floor area of the main activity sports halls usually occupies about half of the floor area of the whole premises (Sports Council, 1981; South Western Council for Sports and Recreation, 1990).

Up to 1981, recreational buildings ranged from 'small community provision' with typically a 20 x 16.9m hall, through small scale and medium scale centres, to the large scale 'wet and dry' sports and leisure centres, with main sports halls typically being of 36 x 32m dimension, in addition to extensive ancillary and specialist provision. Using Sports Council literature (Sports Council, 1981) 23 sports centres were selected as samples; of these eight were found to have a length-width aspect ratio of close to unity. It is also the recent trend and recommendation to design the plan of a sports centre building as close to a square as possible (Scottish Sports Council, 1992). The Standardised Approach of Sports Halls (SASH) presents standardised design data for sports hall construction. Here, the standardised construction recommends that the main sports hall occupies half the building plan area, making the length-width ratio of the building close to unity (Sports Council, 1986). In UK, 27 SASH sports centres have been built in England and Wales. The height of a sports hall is governed by the type of sports / activities. The minimum value is 6.7m for small and 7.6m for other sizes of sports halls. Using data from a variety of sources (Sports Council, 1968, 1986, 1990; Scottish Sports Council, 1992), Table 5.1 summarises the dimensions of the sports halls within the overall premises.
### Table 5.1: Dimensions for different types of sports centres (Sports Council, 1968, 1986, 1990; Scottish Sports Council, 1992)

<table>
<thead>
<tr>
<th>Source</th>
<th>Width</th>
<th>Length</th>
<th>Height</th>
<th>Notes / recommendations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standardised Approach to Sports Halls (SASH)</td>
<td>17m</td>
<td>32m</td>
<td>7.6m</td>
<td>Whole building: 34 × 37m.</td>
</tr>
<tr>
<td>Low Cost Halls (LCH)</td>
<td>17m</td>
<td>32m</td>
<td>7.6m</td>
<td>The stated dimensions are for the activity area only.</td>
</tr>
<tr>
<td>Sports Halls in Rural Towns in Scotland (SHIRTS)</td>
<td>16.5m</td>
<td>26m</td>
<td>6.7m for main sports hall, at least 3.5m for storage area, at least 2.6m for changing areas</td>
<td>Structural dimensions: 25 × 32m; Preferably with longer side facing due north</td>
</tr>
<tr>
<td>Large halls</td>
<td>32m</td>
<td>36.5m</td>
<td>9.1m</td>
<td>Hall area only</td>
</tr>
<tr>
<td>Medium halls 1</td>
<td>26m</td>
<td>32m</td>
<td>7.6m</td>
<td>Hall area only</td>
</tr>
<tr>
<td>Medium halls 2</td>
<td>26m</td>
<td>29m</td>
<td>7.6m</td>
<td>Hall area only</td>
</tr>
<tr>
<td>Medium halls 3</td>
<td>23m</td>
<td>32m</td>
<td>7.6m</td>
<td>Hall area only</td>
</tr>
<tr>
<td>Small halls 1</td>
<td>17m</td>
<td>32m</td>
<td>6.7–7.6m</td>
<td>Hall area only</td>
</tr>
<tr>
<td>Small halls 2</td>
<td>16.5m</td>
<td>29.5m</td>
<td>6.7–7.6m</td>
<td>Hall area only</td>
</tr>
<tr>
<td>Small halls 3</td>
<td>16.5m</td>
<td>26m</td>
<td>6.7–7.6m</td>
<td>Hall area only</td>
</tr>
</tbody>
</table>

Inspection of Table 5.1 shows that 'SASH', 'LCH', 'SHIRTS' and 'small halls' all have the same or similar dimensions. Since these comprise the majority of sports hall designs in the UK, it was described that the 'standard' size of sports hall to be used for simulation will be 32m long 17m wide and 7.6m high. The information discussed also suggested that the sports hall area is to occupy half of the entire plan area of the building. The remaining facilities will include a changing room, a weight-training room and WCs, and will be arranged so as to form as square a building plan as possible. A slope of 10° will be assumed for the profiled metal roof (typical value), with a central ridge.

### 5.3.2. Constructions

The construction of the 'standard' sports centre building model was based on a SASH sports centre recommendation (Table 5.2). Part-L of the Building Regulation (Davis, 1991) provided the basis of U-values for different construction components in the 'standard' model (Table 6.2) since a building that complies with the earlier building regulations is assumed here (a retrofit application).
Chapter 5 - Case study: Use of the proposed solar collector system in a sports centre

**Table 5.2: U-value requirement for sports centre buildings according to Part-L of Building Regulations (Davis, 1991)**

<table>
<thead>
<tr>
<th>Condition</th>
<th>U-value ($\text{Wm}^{-2}\text{°C}^{-1}$)</th>
<th>Walls</th>
<th>Roofs</th>
<th>Floors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double glaze occupying half the total window area</td>
<td>0.60</td>
<td>0.35</td>
<td></td>
<td>0.45</td>
</tr>
<tr>
<td>Double glaze all windows</td>
<td>0.60</td>
<td>0.35</td>
<td></td>
<td>uninsulated</td>
</tr>
</tbody>
</table>

External walls

Two types of external walls were assumed: 100mm common brick wall for the sports hall, running from ground level to 3 metres above ground level, and also for any other zones other than the sports hall; profiled metal cladding for the sports hall running from 3 metres above ground level to 7.6 metres above ground level.

Floor

The ground floor construction was assumed to be of 300mm heavyweight concrete with 100mm insulation. The underneath of the floor was assumed to be exposed to the ground, which was treated as the outside environment.

Roof / Ceiling

Profiled metal cladding was assumed as the roofing material with 100mm glass fibre as building insulation. The intermediate floor (ceiling) was assumed to be of 300mm heavyweight concrete.

Windows/ Rooflight

Both windows and rooflight were assumed to be single glazed (Table 5.3). The area for each window / rooflight was decided according to the maximum area limit as described in Part-L of the Building Regulations (Davis, 1991). It was assumed that sports centres fall under the category of 'other residential buildings'. The Regulations (Davis, 1991) for 'other residential buildings' stated that the maximum single glazed areas of windows / rooflights to that of wall / roof area are 25% and 20%, respectively, though the windows and
rooflights in the 'standard' model are assumed to be double glazed. These figures were used in the 'standard' model for the typical sport centre building.

<table>
<thead>
<tr>
<th>External transparent surface</th>
<th>$\tau$</th>
<th>$U_{\text{glass}}$</th>
<th>$h_i$</th>
<th>$h'_i$</th>
<th>$KL$</th>
<th>Refraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window</td>
<td>0.79</td>
<td>6.11</td>
<td>8.33</td>
<td>16.67</td>
<td>0.08</td>
<td>1.52</td>
</tr>
<tr>
<td>Rooflight</td>
<td>0.79</td>
<td>6.11</td>
<td>8.33</td>
<td>16.67</td>
<td>0.08</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Table 5.3: External transparent surface properties as used in 'standard' model ($\tau$=transmissivity, $U_{\text{glass}}$=conduction heat transfer coefficient through glass, $h_i$=combined convection and radiation heat transfer coefficient between inside glass surface and indoor environment, $h'_i$=combined convection and radiation heat transfer coefficient between outside glass surface and outdoor environment, $KL$=product of thickness and extinction coefficient)

5.3.3. Heating/Ventilation system

In sports halls, a mechanical ventilation system may either be in addition to a radiant heating system, or it may constitute a convective system in its own right (Sports Council, 1981). In either case, the proposed air heating solar collector system can be integrated into the existing heating system in the form of a ventilation pre-heating device.

Gas-fuelled wall-mounted fan-assisted convectors have also been recommended for indoor tennis court heating (Sports Council, 1994), the proposed solar collector system can also be integrated to such a system and fresh air for ventilation can be preheated; this follows the same integration approach as for a radiant heating system.

Recommendations have been made regarding the location and protection of all air input and extraction grilles or openings; the aim here is that the resultant air movement should be such that it minimises disturbance to the flight path of shuttlecocks (Sports Council, 1995). At a small sports centre in Tamworth, Staffordshire, the multi-purpose hall is heated via a hot air plenum discharging air at high level through a fan system in the hall ceiling. Ancillary areas are heated by means of fan convectors; extract ventilation is provided by means of individual fans. The heating system was reported to be effective and the passage of warm air through the grilles was shown not to affect the flight of the shuttlecocks (Sports Council, 1989). This demonstrates that warm air heating by no means unusual in sports halls and leisure centres, and can
be considered in the formation of the 'standard' building for subsequent simulation.

Table 5.4, 5.5 and 5.6 summarise the usual systems and conditions employed in the design of sports centres.

<table>
<thead>
<tr>
<th>Recommendation by &quot;Handbook of sports and recreational building design&quot;</th>
<th>SASH</th>
<th>SHIRTS</th>
<th>CIBSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports hall usually heated by warm air (may be supplied from a central system or from local fan convectors), radiant heating surfaces or a combination of both.</td>
<td>Sports hall and fitness room: 50% recirculation; changing: all fresh air with heat reclaim unit; toilet: all fresh air. Ventilation in each zone above is by an AHU. Other areas are heated by radiators from the main heat distribution circuit. Heat source is typically provided by three 90kW gas fired boilers. The recirculation system in air handling units for hall and fitness room automatically adjusts the amount of fresh air introduced into the space by a system of interlinked dampers so that when the outside temperature is low, fresh air is kept to a minimum and recirculation to a maximum.</td>
<td>Gas-fired central heating with zoned temperature control on radiators. Radiant rather than convective heating was recommended as the heated air will rise to upper volume levels of the hall unused. However, it was thought that a heating system capable of fast response is required.</td>
<td>Refrigeration in the majority of sports centres. May also be required for restaurants, meeting rooms etc.</td>
</tr>
<tr>
<td>Parts of the warm air would have to be fresh air for ventilation. In the case of radiant heating systems, an addition system for ventilating the hall would have to be provided.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radiant heating: Direct fired or fed from central plant. In small halls direct fired warm air or radiant heaters may also be used.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Recommendation by &quot;Handbook of sports and recreational building design&quot;</th>
<th>SASH</th>
<th>SHIRTS</th>
<th>CIBSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports Hall: 16 °C</td>
<td>Sports hall, fitness room: 21 °C; Changing rooms and showers: 25 °C; Offices, circulation and social areas: 21 ºC.</td>
<td>Sports hall: 12–21 °C. Relative humidity may be within the range of 40–90% but should preferably be at about 60%.</td>
<td>Sports hall: 16°C Warmed air to maintain a temperature of 24°C should be provided for changing rooms.</td>
</tr>
</tbody>
</table>

Table 5.5: Recommendation for room temperatures and other environmental conditions in sports centres (CIBSE, 1986; Sports Council, 1981, 1986, 1988a; Scottish Sports Council, 1992)
Chapter 5 - Case study. Use of the proposed solar collector system in a sports centre

Recommendation by "Handbook of sports and recreational building design"

<table>
<thead>
<tr>
<th>SASH</th>
<th>SHIRTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports hall: 1.5 ac/hr. A minimum of 20% and a maximum of 100% of fresh air would be typical.</td>
<td>Hall: 1.5 ac/hr. Changing room: 10 ac/hr</td>
</tr>
<tr>
<td>Sports hall: 1.5 ac/hr, Changing rooms: 10 ac/hr, 100% outdoor air.</td>
<td></td>
</tr>
<tr>
<td>100% outdoor. Lavatories: 10 ac/hr, 100% outdoor. Fitness room: 8 ac/hr, 50% minimum outdoor air.</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.6: Recommendations for ventilation requirements in sports centres (Sports Council, 1981, 1986, 1988a; Scottish Sports Council, 1992)

The heating / ventilation recommendation for SASH centres were adopted for modelling purposes. The SASH recommendation design strategy was chosen because its aim is to be more energy efficient, as well as being representative of typical sports halls in current use.

5.3.4. Occupancy patterns

The opening times of sports centres can vary from one to another, according to the customer profile which is served. Table 5.7 summarises the occupancy patterns of 2 typical sports centre types and an existing sports centre (Tamworth) with different occupant patterns.

<table>
<thead>
<tr>
<th>Days of week</th>
<th>SASH</th>
<th>Low cost hall, Rochford</th>
<th>Tamworth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekdays</td>
<td>0900-2300</td>
<td>1200-2300</td>
<td>1900-2230</td>
</tr>
<tr>
<td>Saturday</td>
<td>0900-2300</td>
<td>1200-2300</td>
<td>1400-1730</td>
</tr>
<tr>
<td>Sunday</td>
<td>0900-2300</td>
<td>0900-2300</td>
<td>0900-1230</td>
</tr>
<tr>
<td>Note</td>
<td>Open 356 days a year. Majority of day time use is by colleges and schools.</td>
<td>Opening hours are subject to alteration.</td>
<td>School holiday afternoons: 1400-1600 (Mon-Fri). The timetable of the sports centre attempts to strike a balance between ordinary bookings, club use, instructional course and activities.</td>
</tr>
</tbody>
</table>

Table 5.7: Opening time of 3 different sports centres types (Sports Council, 1980, 1986 and 1989)

It can be seen that the opening hours of each sports centre can differ. For simulation, the assumption will be for opening time from 1000 to 2200 hours.

To account for heat gain due to occupants, the game of badminton was assumed as the usual activity in the sports hall. Since there was no figure on heat gain for the activity, the figures for occupant's heat generation per person for the game of tennis was assumed. As for the weight training room, the amount of heat generation per person was assumed to be that between
that for tennis and basketball. SASH design guide (Sports Council, 1986) recommended that the number of occupants at any time was to be between 60 and 70 people. However, data on occupancy distribution was not available and that an educated estimation was needed. Table 5.8 and 5.9 summarises the assumptions for heat gain due to occupant and the occupancy distribution for the 'standard' sports centre building.

<table>
<thead>
<tr>
<th>Activities</th>
<th>Sensible heat generation per adult person (W)</th>
<th>Latent heat generation per adult person (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office activity (walking about)</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Calisthenics/exercise</td>
<td>350-470</td>
<td>526-706</td>
</tr>
<tr>
<td>Tennis (singles)</td>
<td>420-540</td>
<td>630-810</td>
</tr>
<tr>
<td>Basketball</td>
<td>580-880</td>
<td>870-1320</td>
</tr>
</tbody>
</table>

Table 5.8: Sensible heat generation per adult body (CIBSE, 1986; ASHRAE, 1993)

<table>
<thead>
<tr>
<th>Area</th>
<th>Day-time (School use)</th>
<th>Evening and weekend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports hall</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>General office</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Bar &amp; snake bar lounge</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Changing room &amp; shower</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>WC</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Fitness room (weight room)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Corridor</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Staff room</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>58</td>
<td>63</td>
</tr>
</tbody>
</table>

Table 5.9: Assumption for number of occupants in the 'standard' sports centre building (at any one time during opening hours)

5.3.5. Lighting and other heat gains

In SASH buildings, the aim is to provide uniform illuminances at floor level of 350 lux for the sports hall, 150 lux for changing rooms, lavatories and showers, 300 lux for the fitness room, and 150 lux for other areas. These figures are the same as those recommended by CIBSE (1986). The lighting throughout the 'standard' sports centre building was assumed to operate from 1000 to 2200 hours.

Table 5.10 summarises these illuminance values, together with the floor dimensions for various rooms that will be used in the simulation. Table 5.11 shows the corresponding estimated heat gains due to lighting that will be employed in the simulation.
Chapter 5 - Case study: Use of the proposed solar collector system in a sports centre

<table>
<thead>
<tr>
<th>Area</th>
<th>Length, (L) (m)</th>
<th>Width, (W) (m)</th>
<th>Mounting height, (H) (m)</th>
<th>Room index, (k)</th>
<th>Illuminance required (Lux)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports hall</td>
<td>32.0</td>
<td>17.0</td>
<td>7.6</td>
<td>1.46</td>
<td>500</td>
</tr>
<tr>
<td>General office</td>
<td>6.8</td>
<td>3.8</td>
<td>3.2</td>
<td>0.76</td>
<td>500</td>
</tr>
<tr>
<td>Bar and snack bar lounge etc</td>
<td>18.0</td>
<td>9.8</td>
<td>3.2</td>
<td>1.98</td>
<td>150</td>
</tr>
<tr>
<td>Changing room &amp; showers</td>
<td>12.0</td>
<td>7.7</td>
<td>3.2</td>
<td>1.47</td>
<td>150</td>
</tr>
<tr>
<td>WC</td>
<td>9.0</td>
<td>4.3</td>
<td>3.2</td>
<td>0.91</td>
<td>150</td>
</tr>
<tr>
<td>Fitness room</td>
<td>10.8</td>
<td>8.0</td>
<td>3.2</td>
<td>1.44</td>
<td>500</td>
</tr>
<tr>
<td>Corridor, staff room &amp; plant access</td>
<td>22.0</td>
<td>3.2</td>
<td>3.2</td>
<td>0.87</td>
<td>150+</td>
</tr>
</tbody>
</table>

Table 5.10: Dimensions and illuminance requirements for each zone

<table>
<thead>
<tr>
<th>Area</th>
<th>Luminarie type assumed</th>
<th>Output of lamp assumed (W)</th>
<th>Illuminance per lamp (lumens)</th>
<th>UF</th>
<th>No. of lamp required</th>
<th>Estimated heat gain due to lighting (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports hall</td>
<td>a</td>
<td>125</td>
<td>9500</td>
<td>0.42</td>
<td>93</td>
<td>11625</td>
</tr>
<tr>
<td>General office</td>
<td>b</td>
<td>125</td>
<td>9500</td>
<td>0.29</td>
<td>4</td>
<td>500</td>
</tr>
<tr>
<td>Bar and snack bar lounge etc</td>
<td>b</td>
<td>125</td>
<td>9500</td>
<td>0.5</td>
<td>8</td>
<td>1000</td>
</tr>
<tr>
<td>Changing room &amp; showers</td>
<td>a</td>
<td>125</td>
<td>9500</td>
<td>0.47</td>
<td>5</td>
<td>625</td>
</tr>
<tr>
<td>WC</td>
<td>a</td>
<td>125</td>
<td>9500</td>
<td>0.32</td>
<td>3</td>
<td>375</td>
</tr>
<tr>
<td>Fitness room</td>
<td>a</td>
<td>125</td>
<td>9500</td>
<td>0.41</td>
<td>15</td>
<td>1875</td>
</tr>
<tr>
<td>Corridor, staff room &amp; plant access</td>
<td>a</td>
<td>125</td>
<td>9500</td>
<td>0.31</td>
<td>5</td>
<td>625</td>
</tr>
</tbody>
</table>

Table 5.11: Internal heat gain due to lighting in each zone (Note: * = area weighted, a = enclosed plastic diffuser, b = suspended opaque-sided luminaire, + = based on minimum requirement, UF = Utilization Factor)

Other sources of heat gain include heat from appliances and solar gain via windows. Heat gain from appliances in a sports hall is usually small compared to other gains and therefore was considered negligible. The amount of heat gain due to passive solar gain via windows can vary from time to time. Therefore, it is necessary to account for solar heat gain only in thermal modelling using a simulation program with a typical meteorological year weather file describing the UK climate. The thermal simulation of the 'standard' sports centre building will be described in the next chapter.
5.4. Summary

In this chapter, a range of applications for the profiled cladding air-heating solar collector system has been identified, ranging from buildings with high fresh air requirements to retail outlets. However, as an 'intermediate', realistic and conservative example, sports / leisure facilities have been investigated. with a view to forming a case study in which to assess the collector system performance. The assessment is to be carried out using a computer-based simulation exercise, and thus requires a typical or 'standard' building upon which to conduct simulations. A literature review of sports buildings has been conducted and reported, and from this a 'standard' sports centre building has been formulated. This is eventually based on the so-called 'SASH' design for sports centres, this being a major source of design guidance for community sports facility provision. The 'SASH'-type sports centre is intended to provide a facility at a comparative low cost, which being capable of accommodating a range of sporting and community uses (Sport Council, 1991a). Furthermore, these types of sports centres account for a substantial percentage of the total facilities in this sector (South Western Council for Sports and Recreation, 1990).

It is assumed that the proposed collector system is to be integrated into an existing sports centre which uses a boiler-supplied convective warm-air ducted heating system (though the collector could be integrated into any building that has one or more profiled-clad facades and a mechanical ventilation system to deliver fresh air). Although the installation is assumed to be for a building retrofit application in this instance, it can be used for new-build projects, also.

In the next chapter, the choice of simulation software is described, the conditions to be investigated are presented, and the results obtained are discussed.
References

ASHRAE (1993), 'Fundamentals Handbook (SI)', ASHRAE, Atlanta


Gibson D. (1996), Private Communication, Gibson Hamilton, Loughborough, 2 July


Jones D. (1992), 'Insulated Solar Collector Wall', Final Year Research Project, Department of Civil and Building Engineering, Loughborough University of Technology


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Sports Council (1985), 'Energy Data Sheet No.4: Ventilation', Sports Council, London


Centre, a Joint Provision Project at Tamworth, Staffordshire', TUS Design Note No.9, Sports Council, London


Sports Council (1991a), 'West Midlands Low Cost Sports Hall initiative'. Sports Council, West Midlands


South Western Council for Sport and Recreation (1990), 'Subject Report - Community Sports Facilities (Sports Halls and Indoor Swimming Pools): Regional Recreation Strategy', South Western Council for Sport and Recreation, February


The Association of Local Authorities of Northern Ireland (1981), 'Local Leisure Centres: Ideas for Provision in the Eighties resulting from an Architectural Ideas Competition'. The Association of Local Authorities of Northern Ireland, March
Chapter 6 - Performance evaluation of the solar collector system by simulation

6.1. Introduction

The previous chapter described the selection of a sports centre as an appropriate building type for evaluating the likely performance of the solar collector system in the UK climate. The evaluation is to be performed by simulation, using a suitable software environment, with the aim of estimating annual energy performances for a sports centre with, and without, the solar collector system. The first part of chapter 6 explains the evaluation and selection of a suitable software environment. This is followed by the preparation of a 'base case' simulation, consisting of the 'standard' sports hall without the solar collection system; this 'base case' is then validated by comparison of predicted energy consumptions with values for actual sports centres quoted in the literature. The next section describes how the solar collector system (in the form of the model presented in Chapter 2) is integrated into the simulation program so as to represent the performance of the solar collector as an integrated facade of the building. The effects of collector orientation, aspect ratio, channel depth and control regime, together with likely surface temperatures, are then assessed by simulation, and design guidance is given.

6.2. Evaluation of simulation environment

Thermal simulation programs are tools used for predicting the energy consumptions and thermal performances of buildings and their HVAC plants. There are many thermal simulation programs available, all performing similar basic tasks. The use of such a simulation program is necessary when the system performance is dynamic, that is, involving thermal storage effects and/or interactions between thermal systems and control systems.

Although the procedures for estimating building energy requirements vary considerably in complexity between programs, they all have 3 common elements (ASHRAE, 1993) that require evaluation:

- space load
- primary equipment energy requirements
Chapter 6 - Performance evaluation of the solar collector system by simulation

- secondary equipment load

Here, 'secondary' refers to the equipment that distributes the heating, cooling, or ventilating medium to conditioned spaces, while 'primary' refers to central plant equipment that converts fuel (gas or electric) energy to the desired heating or cooling effect (ASHRAE, 1993).

In this study, an overall aim is to find out the energy and cost savings resulting from the use of the proposed solar collector system and thus its payback period. It is also necessary to estimate the maximum temperature that the polycarbonate cover system and the profiled steel cladding might experience in order to determine whether the materials used are suitable as potential collector cover and as the solar energy absorber components, respectively.

Since the mathematical model presented in Chapter 2 is to be used within the evaluation, the simulation tool to be selected needs to have the ability to adopt user-written subroutines. Two thermal simulation programs were evaluated so that the most appropriate tool could be selected for carrying out the task.

6.2.1. TRNSYS

TRNSYS stands for TRaNsient SYstem Simulation. It was developed at the University of Wisconsin-Madison and is primarily designed to simulate the transient performance of thermal energy systems. Its main feature is that it relies on a modular approach to analyse thermal energy systems. This approach requires an input file in which the user specifies the components that constitute the system and the manner in which they are connected. TRNSYS is based on the response factor method, which is used to solve algebraic and differential equations that relate to building thermal dynamic behaviour (University of Wisconsin-Madison, 1994).

TRNSYS consists of a 'library' of FORTRAN subroutines for modelling subsystem components, and an executor program that enables the user to simulate the performance of a complete system by simulating the performance of a system of interconnected components. Some of the main
component models contained in the TRNSYS library are listed as follows (University of Wisconsin-Madison, 1994):

- solar collectors (different configurations from that proposed, and for equal front and rear ambient temperatures)
- controllers
- pump (and fan)
- heat exchanger (counter/parallel/cross-flow)
- auxiliary heater (on-off heater with set temperature and deadband)
- heating, cooling, space load and air conditioner
- tee, flow mixer and diverter, damper (flow controllers for air or water), pipe (and duct)
- time dependent forcing functions (permits time varying data to be introduced into simulation, usually periodic)
- solar radiation processor (estimates beam and diffuse radiation on surfaces of any orientation from total radiation on horizontal surface)
- heat pump (water or air source using manufacturers performance data)
- the building (walls, roof, room and basement, with the effect of thermal capacity, infiltration, fenestration, etc.)
- algebraic operations
- quantity integrator (with respect to time)
- data reader (commonly used for reading user-supplied meteorological data)
- collector array shading
- psychrometrics
- load profile sequencer
- output subroutines

If a particular component is not available in the library, a user-defined subroutine can be written and included within the library, provided that they are coded in the same manner as the existing subroutines. This is an important facility, as it permits encodement and inclusion of the collector model from Chapter 2 into the TRNSYS simulation. The resulting mathematical models for these subsystem components represent the subroutines in the TRNSYS program (Klein et al., 1976).

The addition of an interactive graphical interface for creating TRNSYS input files in a user-friendly modelling program (PRESIM) and a building model
construction program for Building Description Files (PREBID) had made TRNSYS a very comprehensive building thermal modelling software (Beckman et al., 1994). With PRESIM, a system may be represented as a schematic drawing with components interconnected to each other. This allows a TRNSYS input file to be created for simulation.

6.2.2. BLAST

The Building Loads Analysis and System Thermodynamics (BLAST) program is a comprehensive program used for predicting energy consumption and energy system performance in buildings. The BLAST program has the following capabilities, as stated by the BLAST Support Office (1994):

- it uses 'rigorous and detailed' algorithms to compute loads, simulate fan systems, and simulate boiler and chiller plants;
- it has its own user-oriented input language and is accompanied by a library which contains the properties of all materials, wall, roof, and floor sections as listed in the ASHRAE Handbook;
- its execution time is brief enough to allow many alternatives to be studied in a reasonable amount of time;
- it is not proprietary (exclusive and restrictive) and therefore open access of its source code is allowed to its users;
- it has the ability to include processed heat source in a simulation.

Though BLAST includes primarily US meteorological conditions, it does provide some weather data for UK sites (Taylor, 1994).

BLAST conducts a simulation in the following way (BLAST Support Office, 1994). The 'Zone Load Simulation' computes hourly loads using the response factor method for a building or zone based on user input and weather data. The 'Air Distribution System Simulation' then uses the computed space loads, weather data, and user inputs (describing an air handling system of the building) to calculate hot water, steam, gas, chilled water, and electrical demands. The 'Central Plant Simulation' uses weather data, the results of the air distribution system simulations, and user input (describing the central plant) to simulate the performance of boilers, chillers, on-site power generating equipment, and then computes monthly and annual fuel and electrical power consumption.
With IBLAST (integrated BLAST), a variant of BLAST, the building air handling systems, and plant simulations, are integrated and performed concurrently. IBLAST also has a link to MODSIM, the basis for the HVACSIM+ program (Park et al., 1986), which would allow users to develop a modular simulation of a solar collector and bypass the BLAST 'parser' (input file syntax analyser for BLAST), which would otherwise have to be modified for air heating solar collector simulation (Taylor, 1994). However, integrated BLAST (IBLAST) is still under development.

6.2.3. Program comparison and selection

BLAST lacks model integration. In other words, simulation of the building, its plant and its air handling systems are not integrated one to another, and are not performed concurrently. Although it was suggested that modular simulation of a solar collector was possible (Taylor, 1994), the method of incorporating a solar collector system by a user-written format is not described thoroughly in sufficient detail.

TRNSYS, on the other hand, has been designed so that user-written subroutines can be integrated with comparative ease. This is of prime concern, because it is necessary to include the solar collector model presented in Chapter 2 within the simulation. Though the TRNSYS library already contains some solar collector subroutines, these are of differing configurations from that modelled in this research and are also for the case of front and rear ambient temperatures being equal.

Assessment based on user-friendliness and ability to adopt user-written subroutines suggested that TRNSYS is the most appropriate tool for this task. The introduction of the graphical interface (PRESIM) to create the model coding also enables construction of files that describe a system of interconnected components (TRNSYS Input Files) more effectively. Thus, in view of these attributes and advantages, TRNSYS was selected for this study as the simulation environment.
6.3. 'Base-case' simulation and validation

Before the effect of the solar collector system on building energy performance can be evaluated, it is first necessary to set up a simulation of the building and its heating system in the absence of the collector system. This is the so-called 'base-case', 'standard', against which the modified building performances can be compared.

A 'standard' sports centre computer model was therefore created based on the recommendations for SASH sports centres as described in chapter 5. Ten building zones were assigned for the model. Each zone in the building was assumed to be heated either by ducted warm-air heating (weight training room, first aid room, changing room and sports hall) or by radiant panels (other zones) (Sports Council, 1986). The building plan (Figure 6.1) was very close to a square (32 m by 35 m). The sports hall model was assumed to occupy approximately half of the total plan area of the building. Figure 6.2 illustrates the 'standard' building model with the areas of walls, floors, ceilings, roofs and windows.
Chapter 6 - Performance evaluation of the solar collector system by simulation

a) Weight/Fitness room

b) First aid room

c) Staff room

d) Changing + showers

e) WCs

f) Circulation area

g) Store

h) Bar + lounge
6.3.1. Detailed data input

Weather file

In order to assess the thermal performance of the building in the UK climate, typical UK weather data was required. The CIBSE Example Weather Year - Kew, United Kingdom (CIBSE, 1986a) was used in this study. The Example Weather Year is the actual weather occurring in a particular year which is broadly representative of the average weather which occurred over a long-term period. The period under examination was for the years 1957-68. The year 1964-65 was found (CIBSE, 1986a) to have a dry-bulb temperature distribution which is closest to the long-term average and hence this was used by CIBSE as the example Weather Year. The data consists of hourly dry-bulb ambient temperatures, together with hourly values of total solar irradiance on a horizontal surface.

Structure

The descriptions of the components used in the computer model of the sports centre building are tabulated in Table 6.1. The thermal properties of the material in each layer of construction are tabulated in Table 6.2.
Chapter 6 - Performance evaluation of the solar collector system by simulation

<table>
<thead>
<tr>
<th>Construction (from inside to outside)</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>External wall 1</td>
<td>COMBRICK100, GLASSFIB</td>
</tr>
<tr>
<td>Internal wall</td>
<td>GlASSFIB</td>
</tr>
<tr>
<td>Floor</td>
<td>Plstr/Gpsm20, LWConc100, HWConc300, Insul.100, Stucco25</td>
</tr>
<tr>
<td>Roof</td>
<td>GLASSFIB</td>
</tr>
<tr>
<td>Ceiling (internal)</td>
<td>Plstr/Gpsm20, HWConc300, Stucco25</td>
</tr>
<tr>
<td>Dummy layer</td>
<td>DUMMY</td>
</tr>
</tbody>
</table>

Table 6.1: Components descriptions of each construction used in modelling. Note the use of the 'dummy layer'; this is used to facilitate the introduction of the building-integrated solar collector (see section 6.4)

Heat gains due to occupancy and lighting (estimated for each zone in chapter 5) were used as the internal heat gain to the computer model.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (m)</th>
<th>Conductivity (Wm⁻¹°C⁻¹)</th>
<th>Capacity (kJkg⁻¹°C⁻¹)</th>
<th>Density (kgm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plstr/Gpsm20</td>
<td>0.020</td>
<td>0.727</td>
<td>0.84</td>
<td>1602</td>
</tr>
<tr>
<td>LWConc100</td>
<td>0.100</td>
<td>0.173</td>
<td>0.84</td>
<td>641</td>
</tr>
<tr>
<td>HWConc300</td>
<td>0.300</td>
<td>1.731</td>
<td>0.84</td>
<td>2243</td>
</tr>
<tr>
<td>Insul.100</td>
<td>0.100</td>
<td>0.043</td>
<td>0.84</td>
<td>91</td>
</tr>
<tr>
<td>Stucco25</td>
<td>0.025</td>
<td>0.692</td>
<td>0.84</td>
<td>1858</td>
</tr>
<tr>
<td>GLASSFIB</td>
<td>0.100</td>
<td>0.04</td>
<td>0.84</td>
<td>12</td>
</tr>
<tr>
<td>COMBRICK100</td>
<td>0.100</td>
<td>0.186</td>
<td>0.8</td>
<td>1500</td>
</tr>
<tr>
<td>DUMMY</td>
<td>-</td>
<td>100000</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.2: Properties of various material used for constructions in computer models (Table 6.1). Note the use of the 'DUMMY'; this is used to facilitate the introduction of the building-integrated solar collector (see section 6.4)

Air flow rates and control settings

The sports hall, weight training room, toilets and changing room were assumed to be warm air heated, whereas the first-aid room, staff room, circulation area, store room, bar and lounge area and general office heated by radiators. The warm air heated areas were assigned as having different
ventilation requirements, environmental conditions according to the requirement (Tables 6.3 and 6.4). Figure 6.3 describes visually the control strategy for heating, ventilation and temperature control strategy in each warm air heated zone.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Heating mode</th>
<th>Heating set point temperature (°C)</th>
<th>Infiltration rate (h⁻¹)</th>
<th>Ventilation rate (kgs⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight training</td>
<td>WA</td>
<td>16</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>First aid</td>
<td>R</td>
<td>21</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Staff room</td>
<td>R</td>
<td>21</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Changing room</td>
<td>WA</td>
<td>25</td>
<td>4</td>
<td>2.51</td>
</tr>
<tr>
<td>WCs (Gents &amp; ladies)</td>
<td>WA</td>
<td>21</td>
<td>1</td>
<td>0.40</td>
</tr>
<tr>
<td>Circulation area</td>
<td>R</td>
<td>21</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Store</td>
<td>R</td>
<td>16</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Bar &amp; lounge</td>
<td>R</td>
<td>21</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>General office</td>
<td>R</td>
<td>21</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Sports hall</td>
<td>WA</td>
<td>16</td>
<td>1</td>
<td>4.53</td>
</tr>
</tbody>
</table>

Table 6.3: Heating mode and settings of different zones in the 'standard' model (Note: WA = warm air heated, R = hot water radiator heated)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Minimum recirculation (%)</th>
<th>Outdoor temperature at which recirculation is at minimum (°C)</th>
<th>Outdoor temperature at which recirculation is at maximum (100% outdoor air) (°C)</th>
<th>Heater battery capacity (kW)</th>
<th>Heat reclaim effectiveness (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight training</td>
<td>50</td>
<td>0</td>
<td>4</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Changing room</td>
<td>100*</td>
<td>-</td>
<td>-</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>WCs</td>
<td>100*</td>
<td>-</td>
<td>-</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>Sports hall</td>
<td>50</td>
<td>0</td>
<td>4</td>
<td>70</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.4: Fresh air requirement and control of recirculated air in warm air heated zones for the 'standard' sports centre building model. (Note: * no recirculation but heat reclaimation, + no recirculation and no heat reclaimation)
The amount of heat from the heater battery (in an air handling unit) to the air stream in each circuit was treated as being controlled by a proportional controller. The use of a proportional-only controller for the simulation was considered adequate as it was the comparative energy consumptions that were of interest in this study. Figure 6.2 shows the temperature control strategy (as well as the air flow control strategy) of the warm air heated zones. The room temperature in each zone was allowed to drift above the room set-point temperature during the warmer seasons (when boiler is not operated and full fresh air is allowed in each zone) since no cooling is usually provided for sports centres in the UK.

**Boiler capacity**

To estimate the maximum power required for boiler output, the heat required in each zone has to be determined. To do this, the maximum possible fabric heat loss in each zone was calculated.
Table 6.5: Maximum possible fabric heat loss in each zone (outdoor temperature assumed: -1°C)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Lenght (m)</th>
<th>Width (m)</th>
<th>Height (m)</th>
<th>Zone temperature set-point (°C)</th>
<th>Fabric heat loss (kW)</th>
<th>Air mass flow rate (kgs⁻¹)</th>
<th>Minimum supply air temperature (°C)</th>
<th>Air temperature before entering air heater (°C)</th>
<th>Maximum heat needed to zone / heater battery (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>12.0</td>
<td>8.0</td>
<td>3.2</td>
<td>16</td>
<td>3.9</td>
<td>1.0</td>
<td>20.3</td>
<td>6.0</td>
<td>14.6</td>
</tr>
<tr>
<td>Sport hall</td>
<td>32.0</td>
<td>17.0</td>
<td>7.6</td>
<td>16</td>
<td>40.6</td>
<td>4.5</td>
<td>26.1</td>
<td>6.0</td>
<td>92.4</td>
</tr>
<tr>
<td>General</td>
<td>6.8</td>
<td>3.8</td>
<td>3.2</td>
<td>21</td>
<td>1.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
</tr>
<tr>
<td>Bar &amp; lounge</td>
<td>18.0</td>
<td>12.0</td>
<td>3.2</td>
<td>21</td>
<td>9.6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>11.7</td>
</tr>
<tr>
<td>Store</td>
<td>16.0</td>
<td>3.8</td>
<td>3.2</td>
<td>16</td>
<td>1.6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
</tr>
<tr>
<td>Circulation</td>
<td>22.0</td>
<td>6.0</td>
<td>3.2</td>
<td>21</td>
<td>4.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.6</td>
</tr>
<tr>
<td>WCs</td>
<td>9.0</td>
<td>4.3</td>
<td>3.2</td>
<td>21</td>
<td>1.8</td>
<td>0.4</td>
<td>25.2</td>
<td>-4.0</td>
<td>11.9</td>
</tr>
<tr>
<td>Changing</td>
<td>12.0</td>
<td>12.0</td>
<td>3.2</td>
<td>25</td>
<td>21.4</td>
<td>2.5</td>
<td>34.3</td>
<td>14.9</td>
<td>49.6</td>
</tr>
<tr>
<td>Staff room</td>
<td>4.0</td>
<td>2.5</td>
<td>3.2</td>
<td>21</td>
<td>8.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.0</td>
</tr>
<tr>
<td>First aid</td>
<td>4.0</td>
<td>3.4</td>
<td>3.2</td>
<td>21</td>
<td>6.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6.6</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>202.8</strong></td>
</tr>
<tr>
<td><strong>add 15%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>233.2</strong></td>
</tr>
</tbody>
</table>

Note that the amount of heat required to the heater battery in each warm-air heated zone is not equal to the fabric heat loss from the same zone. This is because some heat is already lost when fresh air is drawn into and extracted from a zone. Table 6.5 presents the heat requirement to each zone (via either water-filled radiator or heater battery) and the estimated boiler capacity.

The boiler capacity was calculated as follows. The maximum load required by each heater battery (in the case of an air heating zone) and radiator (which equates to the maximum heat loss of the zone) were added together, under the assumption that the outdoor temperature is at -4°C and that temperature in each zone as defined in Table 6.5, and adding 15% of the total maximum heat load requirement, giving the total required boiler capacity of 233kW. Three separate boilers serving the premises were assumed, the load of each boiler being assumed to be a third (78kW) of the total required load (233kW). This may be compared with the SASH design (Sports Council, 1988) which assumes that the heat source comes from a bank of three gas fired boilers, each of 90kW capacity. The part-load performance of each boiler was assumed to be that of a typical domestic boiler, with the part-load characteristic taken from Holmes (1976), as shown in Figure 6.4.
6.3.2. 'Base case' results and validation

A simulation run was then performed with the 'base case' or 'standard' sports centre building, that is, without the solar collector system. The purpose of this was to test the validity of the simulation in comparison with expected behaviour and measured data from actual sports centres.

The time step for the simulation was set to 30 seconds. One reason for this is that the proportional-only control of the heating system was assumed to have the fastest thermal response, despite the fact that the building thermal response would be slower due to its thermal mass. The other reason for choosing the time step is for a better stability of simulation processes.

Figures 6.5a, b, c, d show the air temperatures in three zones (weight training room, sports hall and changing room) as a function of time over a period of seven days in each of the four seasons. Comparison with the set point temperatures stated in Table 6.5a shows that the temperatures in the air-heated zones were being controlled satisfactorily, that is, the simulation is behaving correctly.

Figures 6.5b,c,d show situations when heat gain from occupants, light, and/or solar gain exceed the fabric heat loss, allowing air temperature to drift above set-point. This happens during warmer seasons when no heat is needed in some warm-air heated zones as explained earlier.

Figure 6.4: Part-load characteristic of boilers (Holmes, 1976)
The weekly total energy consumptions over the year as estimated from the simulation are presented in Figure 6.6. This shows the expected reduction in heat delivery during the summer period. To further test the accuracy of the 'standard' sports centre simulation model, the estimated gas consumptions to heat different zones of the building as predicted from simulation were compared with published data (Table 6.7) (Sports Council, 1988). For the five 'SASH'-designed sports centres in Table 6.7, their mean annual gas consumption was found to be 459558 kWh. This may be compared with a value of 454479 kWh as predicted by simulation for the 'standard sports centre model. The agreement is within 1.1% and is further evidence of the satisfactory performance of the 'base case' or 'standard' simulation model.

Figure 6.6: Estimated heating requirement (simulation) by the boilers (south orientated)
In the simulated 'standard' sports hall, the largest vertical facade of the building was assumed to be clad with profiled steel. Note that the largest vertical facade of the 'standard' sports hall consists of brick work (from the ground level to 3.0 m above ground) and profiled cladding (from 3.0 m above ground to 7.6 m above ground, i.e. 4.6 m high). In the simulated 'standard' sports hall, it is also assumed that windows are included only in the east and west-facing vertical facades of the sports hall (for the building orientated south). This corresponds to design practice for 'SASH' centres, so as to minimise glare in the sports hall. Solar gain through these windows can be expected to contribute to the heating of the building, offsetting the energy (assumed gas) consumption required to achieve the desired room temperature set-points. In order to further test the performance of the 'standard' simulation model, the orientation of the building was changed from east through south and then to west-facing, and the total annual building energy consumptions as predicted by simulation were rated (see Table 6.8).

<table>
<thead>
<tr>
<th>Building orientation (direction of large opaque profiled facade)</th>
<th>E</th>
<th>SE</th>
<th>S</th>
<th>SW</th>
<th>E</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glazing orientation (on 2 facades in sports hall)</td>
<td>N, S &amp; NE, SW</td>
<td>E, W &amp; SE, NW</td>
<td>N, S &amp; W &amp; NW</td>
<td>N</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>Predicted annual energy consumption (kWh)</td>
<td>451127</td>
<td>454556</td>
<td>457350</td>
<td>455988</td>
<td>453375</td>
<td>454479</td>
</tr>
</tbody>
</table>

Table 6.6: Predicted annual consumption versus orientation for the simulated 'standard' sports centre (for building Orientation see Figure 6.7)

Figure 6.7: Reference of building orientation
Inspection of Table 6.6 shows that the lowest annual energy consumptions coincide with glazed areas on the sports hall facing south (providing the most solar gain), with the greatest annual energy consumption being required when glazing faces east and west. Energy consumptions for off-compass orientations lie between the previous limits. Note that the difference in energy consumptions when the building is orientated east and west is due to an asymmetry in the Kew irradiance data (see section 6.5.2) and the different heating arrangement of the weight training room (warm-air heated) and the bar-lounge area (heated by hot water-filled radiator).

<table>
<thead>
<tr>
<th>SASH centre</th>
<th>Gas consumptions (kWh)</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barnsley</td>
<td>567491</td>
<td>19 Nov 1984 ~ 18 Nov 1985</td>
</tr>
<tr>
<td>Willington</td>
<td>482954</td>
<td>19 Nov 1984 ~ 18 Nov 1985</td>
</tr>
<tr>
<td>Southampton</td>
<td>327265</td>
<td>19 Nov 1984 ~ 18 Nov 1985</td>
</tr>
<tr>
<td>Colne</td>
<td>379425</td>
<td>12 Nov 1984 ~ 11 Nov 1985</td>
</tr>
<tr>
<td>Shirebrook</td>
<td>490106</td>
<td>19 Nov 1984 ~ 18 Nov 1985</td>
</tr>
<tr>
<td>Mean:</td>
<td>449448</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.7: Reported annual gas consumptions for SASH sports centres in the United Kingdom (Sports Council, 1988)

Table 6.8 Shows a percentage breakdown of the predicted total annual energy consumption by zone of the 'standard' sports centre simulation model. In Table 6.8 these values are compared with figures predicted by a thermal model developed within the Department of Energy's 'Factory Heating Target Project' (FHTP) (Sports Council, 1988) for the four warm-air heated zones, assuming for a SASH centre in Southampton. It can be seen that the percentages for FHTP model (90%) and predicted (83%) energy consumptions for the four warm-air heated zones are in good agreement, together with the figures for the fitness room. While the rank orders as shown in Table 6.8 are also in broad agreement, these is, however, a discrepancy in the measured and predicted percentage for the changing room and for the sports hall.

<table>
<thead>
<tr>
<th>Air-heating zones</th>
<th>FHTP Southampton (%)</th>
<th>Measured ranking</th>
<th>Simulation (%)</th>
<th>Predicted ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports hall</td>
<td>37.0</td>
<td>1</td>
<td>23.0</td>
<td>2</td>
</tr>
<tr>
<td>Toilet</td>
<td>24.3</td>
<td>3</td>
<td>12.0</td>
<td>3</td>
</tr>
<tr>
<td>Changing area</td>
<td>26.3</td>
<td>2</td>
<td>45.0</td>
<td>1</td>
</tr>
<tr>
<td>Fitness room</td>
<td>2.4</td>
<td>4</td>
<td>3.0</td>
<td>4</td>
</tr>
<tr>
<td>Total:</td>
<td>90.0</td>
<td></td>
<td>83.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.8: Comparison of measured and predicted distributions of energy consumption for four warm-air heated zones
The reason for this discrepancy is considered to be due to the way which FHTP calculation accounts for the solar gain. It is worth mentioning that FHTP calculation of space heating energy for a building is considered limited in accuracy since it uses considerably simplified input data (Sports Council, 1988). The other possibility for the discrepancy might be that an air flow rate different from the one used in the simulated changing room was used in the actual changing room, causing the simulated changing room to have a higher heating requirement. The reason for using such a high air flow rate to the changing room was that maximum zone temperature would be reduced to a reasonable level. The same reason was applied for the use of a higher air flow rate to the sports hall. Note that the changing room in such a building can be expected to have a significant energy requirement due to its higher air temperature set-point and the fact that it received a 100% fresh air supply. However, the simulation overpredicts this requirement because the simulated changing room was assumed to be situated on the north side of the sports centre model. Nevertheless, this discrepancy was not considered to invalidate the assessments which follow, due to their comparative nature. In absolute terms, the assessment of the building performance with the solar collector system installed might be considered slightly less conservative, but it must be borne in mind that such a building was never intended to be the ideal 'test-bed' for such a system (buildings with large fresh air requirements would be a better, more 'optimistic', test). Therefore, it was decided that the proposed air-heating solar collector system should provide the pre-heated air supply to the changing room heating circuit, in order that the collected heat may be utilised as much as possible.

In summary, from the preceding tests, it is concluded that the 'standard' sports centre simulation model provides an acceptable representation of the performance of such a building, and can thus be used as a validated 'base case' against which comparisons can be made.

6.4. Integration of the solar collector system to the simulation model

In order to assess the performance of a sports centre equipped with a retrofitted solar collector system, it is necessary to integrate the collector model within the simulation environment. The collector is assumed to become either all (or part) of the existing large profiled steel facade, or all (or part) of one
side of the existing opaque profiled steel roof (pitch of 10°). The integration was achieved firstly by encoding the collector model presented in Chapter 2 so as to form a new component model for addition to the TRNSYS component library. Secondly, it then becomes necessary to connect the new component model into the TRNSYS system simulation such that it is behaves as a wall or roof-integrated building element.

To integrate the collector as part of the structure, a 'dummy' layer (see Tables 6.1 and 6.2) was created in the sports centre computer model which allows heat gain (from an adjacent room to the zone, for example) to be accounted for as an additional heat gain (as would be gained from the solar collector rear surface) to the zone. To model the interaction between the collector and the building, the collector rear heat loss was taken as the additional heat gain to the building (through a 'dummy' layer) and, at the same time, the zone temperature was taken as the collector's rear environment temperature (Figure 6.8). The radiative and convective heat transfer mechanism between the solar collector and the zone, in which the solar collector is integrated, has already been modelled and accounted for within the mathematical model of the collector (as derived Chapter 2) so that the 'dummy' layer does not have to account for this mechanism. Thermal capacitance of the solar collector was found to be small (as estimated in Chapter 3) and therefore considered negligible. Although no thermal capacitance in the solar collector was assumed, the effect of thermal capacitance in the building was modelled, as they were accounted for by each building component in the multi-zone building model already available in the TRNSYS environment.
The solar pre-heated fresh air was considered to be supplied to the changing room only via its air handling unit (AHU). It is necessary to assume a control strategy for the operation of the solar collector. As an initial strategy, the solar collector was considered to operate when all the following conditions were fulfilled:

- irradiance level was above 0 Wm\(^{-2}\)
- outdoor ambient air temperature was below 14°C
- the time of day was within working hours of the sports centre (1000 hour to 2200 hour, as stated in Chapter 5)

These settings were only an initial suggestion and might not be the best combination for the most effective collector operation. Note that the irradiance level setting control enables the collector to be operated only when the irradiance reaches a certain level; an appropriate choice of this setting might encourage more energy efficient operation by the solar collector. Also, the outdoor temperature setting disables the solar collector operation when heat from the collector is not needed, that is, outdoor air temperature is sufficiently high to be supplied directly to the zone. While the collector operation is disabled, the fresh air was drawn directly from outdoors so as to reduce the risk of overheating and thus minimise unnecessary fan operations which would reduce system efficiency. The effects of trying different irradiance level and temperature settings will be presented later.

Figure 6.8: Coupling between proposed air-heating solar collector and the building
A collector absorber-cover spacing of 0.10m was used initially in the simulation. This spacing was used when investigating the effects of collector orientation, inclination and aspect ratio. The absorber-cover spacing is an independent factor when comparing performances between different orientations, inclinations and aspect ratios (length : width) of the collector. The effect of the absorber-cover spacing on performance will be presented later.

6.5. Effect of collector orientation and aspect ratio

6.5.1. Conditions investigated

The collector system was considered to be attached either to one wall of the sports centre, or to one side of its pitched roof. In each case, four different areas were considered. For the case when the collector was attached to the roof, two different aspect ratios (length x width) were investigated, the area being kept constant. In all cases, the orientation of the collector system was varied from west through south to east. The detailed conditions simulated were as follows:

Wall

The assessment was conducted with the proposed solar collector integrated onto the opaque profiled steel clad wall with the largest wall area. The four collector areas to be assessed were of dimensions 32m, 24m, 26m and 8m wide; all were for the same length (height of collector, namely 4.6 m). These dimensions represented 100%, 75%, 50% and 25% of the profiled steel clad facade area, respectively. Note that the profiled steel clad wall on the sports hall is assumed to be from 3 m to 7.6 m above ground level and it is only a portion of the whole facade (the facade consists of profiled steel clad portion as well as a brick lower portion). Note also that the solar energy retrofit only make use of the profiled-clad portion of the whole facade.
Chapter 6 - Performance evaluation of the solar collector system by simulation

Roof1

The assessment was conducted with the solar collector integrated onto one side of the pitched roof of the sports hall. The four collector areas to be assessed were of dimensions 32m, 24m, 16m and 8m wide; all were for the same length of collector, namely 4.6 m. These dimensions represent 100%, 75%, 50% and 25% of the south facing profiled steel clad facade area used and the areas examined are the same as the wall. This was arranged so that direct comparison of performance between the collector system attached to the wall or to the roof might be made. Note that the dimensions given above all related to a collector length of 4.6 m, which will be designated 'roof1' in the results.

Roof2

The assessment was conducted again with the solar collector integrated onto one side of the pitched roof. However, the collector areas to be assessed here were of dimensions 17.06m, 12.79m, 8.53m and 4.26m wide; all were for the same (new) length of collector, namely 8.63 m. Note that these dimensions give the same collector surface areas as those of 'roof1', but now permit direct comparison between different aspect ratios. These are designated as 'roof2' in the results.

6.5.2. Results

Wall results

Figure 6.9 shows the whole-building annual gas consumption by the boiler as a function of the facing direction of the collector system, for four collector areas attached to the wall. Also shown for comparison is the case with no collector system at all (the 'standard' or 'base case' building). Figure 6.10 shows the corresponding gas savings in kWh, brought about through use of the collector system.
Chapter 6 - Performance evaluation of the solar collector system by simulation

The results show the energy-saving effect offered by the addition of the collector system. This energy saving comes about not only through solar collection, but also as a result of improving the wall U-value by the presence of the (retro-fitted) transparent cover system and air flow channel, as well as the 're-cycling' of heat loss from the building through that section of the facade. Not surprisingly, the south orientation gives the largest gas savings, followed by southwest, southeast, west and east (in this order). Also, the larger the collector area, the greater are the gas savings. Note that the simulation assumes the same total constant mass flow rate of air through

\[ \text{Energy savings (kWh) versus orientation of collector facade (wall)} \]

\[ \text{Gas consumption by boiler (kWh) versus orientation of collector facade (wall)} \]

Refer to Table 6.6 for building orientation.
each of the collector systems (2.51 kgs\(^{-1}\)), the value being determined from the amount of air requirement for bringing the zone temperature to an acceptable level during warm season. This implies that as the collector area increases, the air speed in the flow channel reduces (for constant total mass flow rate). The expected mass flow rate ranges from 0.314 kgs\(^{-1}\) per metre collector width (collector 4.6 m long by 8 m wide) to 0.078 kgs\(^{-1}\) per metre collector width (collector 4.6 m long by 32 m wide). However, the range of mass flow rate that the collector test section was tested for ranges from 0.134 kgs\(^{-1}\) to 0.203 kgs\(^{-1}\); figures correspond to air velocities of 2.41 ms\(^{-1}\) (corresponds to Reynolds number, Re, of 30279) and 0.60 ms\(^{-1}\) (corresponds to Reynolds number, Re, of 7626). Since the mathematical model had been validated under turbulent channel flow regime (Re>4000), the model is considered valid to describe these conditions.

Figure 6.11: Gas consumptions for a) 'base case' sports centre model, b) 'base case' sports centre without windows, and c) as b) with 'weight training room' temperature set-point at 21°C
Further inspection of Figures 6.9 and 6.10 shows that gas savings appear greater for the southwest-orientated collector system as compared with the southeast orientation. This was thought to be the result of one or more of the following: a radiation asymmetry in the observed irradiances at the Kew site; the location of the windows in the simulated building, leading to the same passive solar gain which offset gas requirements; the choice of the temperature set-points in the weight training room and the bar-lounge area (the higher the set-point of a zone, the greater the benefit from solar gain to that zone as a result of offsetting the gas requirements). Each of these possibilities was investigated, by plotting the annual incident irradiance profile from the weather file (Figure 6.12) and by carrying out further simulations involving 'blocking' of windows and changing of set-point temperature. The results showed that all these effects were contributing to the difference in gas consumption and savings for southeast / southwest orientation as observed in Figure 6.11a, b and c. It was thus concluded that the simulation was behaving correctly, and that it was suitable for comparative estimation of collector thermal performance.

**Roof results**

Here, the same four collector dimensions and areas (as for the wall results) were considered to be on the roof pitch of 10° to the horizontal. The collector thermal model accounted for the change of orientation by using different convection correlations to replace those that apply in the vertical position. The effects of area and orientation on gas consumption are shown in Figure.
6.13 While Figure 6.14 shows the gas savings for roof collection in comparison with the case of wall correlation.

![Figure 6.13: Gas consumption by boiler (kWh) versus orientation of collector facade (roof)](image)

![Figure 6.14: Gas savings (kWh) versus orientation of collector facade: comparison of wall and roof locations](image)

In general, the same trends regarding collector area and orientation can be seen in Figure 6.13 as compared with Figure 6.9. However, Figure 6.13 shows that, in general, gas savings are increased by setting the collector system on the roof rather than on the vertical facade. Again, this is not an unexpected result, due to the incidence angle of the solar irradiance. However, it suggests that, in this case (Kew location), care should be exercised in choosing between wall or roof-mounting for a south-easterly
orientation; as regards design guidance, it is clearly necessary to consider local effects upon incident irradiance. This suggests operating the simulation design tool with local weather data, if available. In addition, the shallower form of the roof-related plots confirms the expected result of less orientation dependence effect of solar incidence angles as compared with the wall-related plots (Figure 6.9). This again increases confidence in the simulation, together with its new component model, as a potentially useful design tool for building with facade-integrated solar collection systems.

Rooft1 results

The same four collector areas were again investigated as a roof-mounted system, but for a different length to width aspect ratio. Here, the length of the collector system was increased from 4.6 m to 8.3 m (the full length from eaves to ridge), with a corresponding adjustment to collector width so as to maintain a constant collector area. Figure 6.15 shows the gas savings for the two different aspect ratios (roof-mounted).

![Figure 6.15: Gas savings (kWh) versus orientation of collector facade: comparison of aspect ratios (roof-mounted)](image)

Figure 6.15: Gas savings (kWh) versus orientation of collector facade: comparison of aspect ratios (roof-mounted)

The trend is the same as before as regards collector area and orientation. However, increasing the length of the fixed area collector system can be seen to improve the energy savings. This is because the same air mass flow rate is being drawn through a rectangular duct of decreased width, thereby raising the air speed in the duct. The raised air speed increases the values of the convective heat transfer coefficients in the collector duct, leading to improved
heat removal from the collector. (Note: the effects of duct geometry on fan power requirements are discussed later). This serves to further demonstrate the satisfactory behaviors of the collector model and building simulation.

In terms of energy collection, it therefore appears better to raise the air speed in the collector flow channel. This can be achieved by increasing collector length while reducing collector width. A further method, that of reducing the depth of the air flow channel, is discussed next.

6.6. Effect of collector channel depth

In the previous section, it was shown that as the air speed increases in the collector flow channel, more useful heat is collected due to the increase in value of convection coefficients, which in turn gives better heat removal efficiency. All the previous simulations have considered a fixed channel depth (profile 'crest' to cover underside distance) of 0.10 m. Variation of channel depth was therefore investigated next. Previous studies (Choudhury and Garg, 1991) have shown that as the depth reduces and the air speed increases (for a fixed mass flow rate), the rate of energy collection increases (for the reason already given). However, the fan power needed to draw the air through the collector will also rise, because of the increased flow resistance of the narrowing channel. The effect on annual energy savings of the simulated building was therefore investigated for channel depths ranging from 0.10 m to 0.01 m. The simulation was carried out for one roof-mounted collector system of dimensions 8.63 m long by 32 m wide, orientated towards the south. Figure 6.15 shows the results.

It can be seen (Figure 6.16) that as the channel depth reduces, the energy savings increase directly. Thus as far as energy saving is concerned, narrow channels are better.
Figure 6.16: Total annual thermal energy savings versus channel depth; roof-mounted collector system 8.63 m long by 32 m wide, orientated south

However, it is important to also consider the effect on fan power requirements. For each channel depth, these were estimated by the following equations (CIBSE, 1986b):

\[
\frac{1}{\sqrt{f}} = -4 \log_{10} \left[ \frac{e}{3.7d_h} + \frac{1.255}{Re\sqrt{f}} \right] \quad \ldots (6.1)
\]

\[
\Delta p = 4f \left( \frac{1}{d_h} \right) \left( \frac{\rho v^2}{2} \right) \quad \ldots (6.2)
\]

\[
P = m\Delta p \quad \ldots (6.3)
\]

where:

- \(d_h\) = hydraulic diameter of collector channel (m)
- \(e\) = surface roughness of collector channel (m)
- \(f\) = friction (-)
- \(m\) = mass flow rate (kgs\(^{-1}\))
- \(P\) = Fan power (W)
- \(Re\) = Reynolds number of fluid in collector channel (-)
- \(v\) = velocity of fluid in collector channel (ms\(^{-2}\))
- \(\rho\) = density of fluid in channel (kgm\(^{-3}\))
- \(\Delta p\) = pressure drop across collector channel (Pa)
The pressure loss across the associated ductwork (excluding loss across the collector system) was estimated using HEVACOMP. The design and resulting pressure loss estimation is presented in Appendix A10.

Using the pressure loss, together with the pressure loss estimated using the Equations 6.1, 6.2 and 6.3, a relationship between electrical energy consumptions and channel depth was plotted. The effect of channel depth on fan energy consumption is presented in Figure 6.17.

![Figure 6.17: Fan electricity requirement versus channel depth; roof-mounted collector system 8.63 m long by 32 m wide, orientated south](image)

Combining the results of Figure 6.15 with those of Figure 6.16, the net annual energy saving as a function of channel depth is obtained (Figure 6.18).

It can be seen that the interaction between the annual fan energy consumptions and the collector thermal energy savings resulted in an optimum channel depth of 0.01 m. This is due to the relatively lower fan electrical energy consumptions for channel depths between 0.03 m and 0.10 m. As channel depth reduces from 0.03 m to 0.01 m, the magnitude of the annual electrical energy consumed by fan increases intensively. This results in an optimum channel depth of 0.01 m for best energy savings.

Since fan operation (which consumes electricity) is needed to operate the solar collector (which saves gas), the evaluation of the cost saving is not merely the matter of multiplying the net energy savings to the cost of the
energy saved, as would be the case when the same fuel was saved (by the collector system) and consumed (by the fan). Here, both the prices of gas \((1.375 \text{ pkWh}^{-1}; \text{Roberts, 1998})\) and electricity \((5 \text{ pkWh}^{-1}; \text{Gill, 1998})\) to such a premises was considered.

![Net energy benefit versus channel depth; roof-mounted collector system 8.63 m long by 32 m wide, orientated south](image)

Figure 6.18: Net energy benefit versus channel depth; roof-mounted collector system 8.63 m long by 32 m wide, orientated south

Figure 6.19 presents the net cost savings due to the operation of the solar collector system. As can be seen, the maximum annual net cost saving due to the collector system operation occurs at channel depth of 0.02 m. Note that this optimum depth is different from the one for best net energy savings. This is not unexpected since the cost of a unit of electricity is 3.6 times that of gas. This causes the 'turning point' to shift toward the right hand side of the graph (Figure 6.19).

Therefore, accounting for energy collection and fan energy consumption, the optimum channel depth for the solar collector system (profile 'crest' to the underside of cover) has been found to be 0.02 m. This depth is used for the investigation presented in the next section.
6.7. Effect of collector control regimes

6.7.1. Irradiance threshold setting

Throughout the previous simulations, an irradiance level control set-point of 0 Wm$^{-2}$ was assumed. This means that, whenever the outdoor irradiance level exceeded 0 Wm$^{-2}$, air was drawn through the collector, causing the collector to provide energy for the building. In the simulation, the irradiance setting is adjustable, allowing the effect of settings greater than 0 Wm$^{-2}$ to be investigated. Thus, at times when collector operation was disabled (irradiance at or below a chosen threshold level), no flow through the collector takes place, and the collector system is treated and modelled as having an unventilated cavity. The mathematical model for the collector as an unventilated cavity is presented in Appendix A8. In this circumstance, a reduced fabric heat loss from the building will still occur due to the reduced U-value offered by the collector construction. The effect of irradiance threshold setting upon building performance was simulated for the case of 50 Wm$^{-2}$, 100 Wm$^{-2}$, 150 Wm$^{-2}$ and 200 Wm$^{-2}$ for the case of the collector mounted on the roof. The collector dimensions used for this investigation was 8.63 m long by 32 m wide. This area represents the full south-facing profile-clad roof area. The channel depth used for this investigation was 0.02 m.

Figure 6.20 shows the building total annual gas consumption (and the total annual hours of fan operation) as a function of irradiance threshold setting. As
the threshold setting is raised, annual gas consumption rises as that collector operation is disabled for increased periods of time. There is a corresponding fall in the annual number of hours of fan operation.

In order to select an appropriate threshold setting, it is necessary to consider gas consumption (by the boiler), together with the electricity consumption (by the fan). Figure 6.20 and 6.21 show, respectively, the net annual energy consumption, and the net annual cost savings as a function of irradiance threshold setting.

Figure 6.20: Annual gas consumptions versus irradiance threshold setting; roof-mounted collector system 8.63 m long by 32 m wide, orientated south

Figure 6.21: Net annual cost savings (together with fan running cost and cost savings on gas) versus irradiance threshold setting; roof-mounted collector system 8.63 m long by 32 m wide, orientated south
Inspection of Figure 6.20 and 6.21 show that the optimum irradiance settings for both energy saving and for cost saving are 0 Wm\(^{-2}\). This also further demonstrates the utility of the simulation model as a design aid, since other values and prices can be used as inputs, depending upon the prevailing situation.

6.7.2. Outdoor air temperature setting

Through the previous simulation, an outdoor air temperature set-point of 14°C was assumed. In other words, whenever the outdoor air temperature exceeded 14°C, fresh air was drawn directly from outdoor without pre-heated by the collector (by drawing air through it). In the simulation, this temperature setting is adjustable, allowing the effect of temperature settings other than 14°C to be modelled. Thus, at times when collector operation was disabled (outdoor air temperature above a chosen threshold temperature), no flow through the collector takes place, the collector system is modelled as having an unventilated cavity. A reduced fabric heat loss from the building will occur (just as in the previous investigation, section 6.7.1). The effect of outdoor air temperature threshold setting upon building energy performance was simulated for the case of 12°C, 13°C, 15°C, 16°C, 17°C and 18°C. Irradiance level control set-point of 0 Wm\(^{-2}\) was assumed in this exercise.

![Graph showing gas consumptions versus outdoor air temperature threshold setting](image)

**Figure 6.22:** Gas consumptions (and fan operation hour) versus outdoor air temperature threshold setting; roof-mounted collector system 8.63 m long by 32 m wide, orientated south
Chapter 6 - Performance evaluation of the solar collector system by simulation

Figure 6.23: Maximum changing room temperature versus outdoor air temperature threshold setting; roof-mounted collector system 8.63 m long by 32 m wide, orientated south

Observation from Figure 6.22 and 6.23 show that as the outdoor air temperature set-point increases, the gas consumptions decreases. Also, this causes the maximum air temperature of the simulated changing room to increase, resulting in possible thermal discomfort. A maximum changing room temperature of about 39°C was estimated for an outdoor air temperature threshold set-point of 18°C. This maximum changing room air temperature was estimated under the assumptions that all the windows and doors are closed, as assumed in the case of the simulation. The maximum changing room temperature would however be greatly reduced by opening of windows in the room to encourage cooler air to reduce the zone temperature by natural ventilation.

Therefore, for evaluation of the cost effectiveness (payback period, see next chapter) of the solar collector system, an outdoor air temperature threshold set-point of 18°C is used.

Figure 6.24 presents the effect of outdoor air temperature threshold set-point on the annual net cost savings for heating by gas and electricity. As can be seen, the simulated annual net cost savings for using gas as heating fuel is £693.33, compare with £1633.07 for using electricity as heating fuel. When an outdoor air temperature threshold setting of 18°C is used. These figures are used for the evaluation of payback period in the next chapter.
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Figure 6.24: Annual net cost savings versus outdoor air temperature threshold set-point; roof-mounted collector system 8.63 m long by 32 m wide, orientated south; irradiance level threshold set-point 0 Wm\(^{-2}\)

6.8. Absorber and cover temperatures

In this research, both profiled steel and the multi-layered polycarbonate cover material are being proposed for use in a novel manner. In addition to the energy performance of the building, it is also important to know whether surface temperatures within the collector system are likely to exceed any recommended limits as stated by the manufacturers during their operation as a building-integrated solar collector. The model derived in Chapter 2 is able to predict surface temperatures throughout the collector and so a further simulation was conducted with these temperatures as an output. The condition simulated was that of a roof-mounted collector, of dimensions 8.63 m long by 32 m wide, and orientated south. Figure 6.25 shows a section of the output from the simulation over a year, indicating that the cover material and the profiled steel metal absorber may reach temperatures of 131°C and 159°C, respectively. These temperatures exceed the reported maximum 'continuous operating temperature' of the Pvf2 coating (120°C) (British Steel, 1988), and the 'deflection temperature' of the polycarbonate cover (132°C) (GE Plastics, 1997)
Figure 6.25: Estimated cover surface temperature roof-mounted collector system 8.63 m long by 32 m wide, orientated south; irradiance level threshold set-point 0 Wm\(^{-2}\)

These results clearly show that for the practical operation of such a system it will be necessary to provide some form of venting for the collector air channel during times of high irradiance when, for example, pre-heated air is not required within the building. This flow could either be provided by natural convection (if sufficient) or by forced convection. It will, of course, be necessary at such times to supply fresh air to the building directly from outdoors. These modes of operation have not been included within the mathematical modelling as currently presented. However, the model could be modified to accommodate such operation modes.
6.9. General design guidance - energy collection

The simulations conducted have been shown to correctly predict the expected trends as regards the energy performance of the building and collector system; these findings are therefore considered to validate the simulation program together with its new collector component model, in this context. The simulation can thus be used as a design tool for buildings that incorporate such envelope-integrated collector systems. Whilst acknowledging the fact that specific building performances can vary, and that they should be assessed on an individual basis, nevertheless it is considered appropriate to offer some general design guidance at this stage drawn from the preceding results for the case study building in question. Note that the simulation program can be used to conduct many more parametric studies and thus refine any guidance being offered. However, the guidance given below is based only on the conditions investigated so far.

If the retrofitting of an existing building in the UK with standard backing insulation is being contemplated, the following points should be borne in mind, as regards maximising annual energy collection for space heating:

1) Use a facade which faces as close to south as possible.

2) Depending on local surroundings, attach the collector system to the roof, rather than to the vertical facade; (this may also be preferable from the points of view of vandalism risk and glare to passers-by, motorists, etc.).

3) The area of the collector system to install will depend upon the utilisation (Simonson, 1984) of the collected energy by the building (i.e. appropriately match the building load to the supply).

4) Heat removal efficiency from the collector can be maximised by increasing the airspeed through the collector flow channel. The latter can be achieved by reducing the gap between profiled steel surface and the underside of the transparent cover. Additionally (or alternatively), for a given area, the aspect ratio should be selected to maximise the length of air flow. Note, however, that fan power requirements should simultaneously be considered (net cost benefit).
5) The setting of the irradiance level threshold for the control of the current collector system was 0 Wm\(^{-2}\). This threshold setting disabled the collector operation when the irradiance level on the collector surface reduces to 0 Wm\(^{-2}\). In the simulation exercise conducted above, this setting gave the best cost saving than those of higher threshold settings. Concerning outdoor air temperature threshold set-point, a highest possible set-point should be used, provided the temperature of the zone (to which the solar pre-heated fresh air is delivered) can be regulated (by opening of windows, natural ventilation, etc.) to a reasonable level without causing thermal discomfort.

6) It will be necessary to consider the inclusion of some form of flow channel venting system, so as to prevent excessive collector surface temperatures during periods of high solar irradiance.

7) The simulation program, together with its new 'facade-integrated collector' component model, should be used to compare alternative design solutions in more detail, on an individual case basis. The cost-effectiveness of different solutions can then be subsequently evaluated.

6.10. Summary

In this chapter, the selection of a suitable simulation environment has been described, one which is capable of accepting the collector thermal model derived in chapter 2. This resulted in the choice of TRNSYS, and its use as a means for evaluating the energy performance in the UK climate, of a building with the collector system as an integral part of its facade. The integration of the collector, model to the building model, within the simulation software, has been described, thereby producing a new component model for use within the TRNSYS library.

Simulation of a standard 'base case' building (no collector attached) was carried out, and its performance was found to generally agree with published performance. Using the program, a parametric study was then carried out for the case of the same building, but with the solar collector system as an integral part of a facade. The effects of collector area, orientation, and aspect ratio were investigated, together with channel depth and control regime. Results were in agreement with expected trends, further validating the use of the simulation program as a design tool in this context. Though not an
exhaustive set of parametric simulations, the results have served to formulate some general design guidance regarding annual energy performance of such a building (a sports centre); however, for more detailed evaluation of design alternatives for different buildings, it is recommended that the simulation program is used together with the new, facade-integrated collector component model (with standard levels of backing insulation for the collector). The cost-effectiveness of a particular solution can then be subsequently evaluated. The latter aspect is described in the next chapter, for the case study in question.
Chapter 6 - Performance evaluation of the solar collector system by simulation

References


British Steel (1988), 'Product Data Sheet - Colorcoat Pvf2', British Steel, September


Gill R. (1998), Private Communication, 15th February

Holmes M. (1976), 'Partload efficiency of gas and oil-filed boilers', BSRIA, TN 1/76


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Chapter 7 - Economic assessment of the proposed solar collector system

7.1. Introduction

New technologies have seldom emerged in the marketplace unless they were shown to be economically competitive with existing technologies (Kreider and Kreith, 1981). Solar energy technology is no exception. Solar energy (electrical or thermal) is still a developing technology that requires reliable estimates of cost-effectiveness over the lifetime of a given system. Frequently, the 'payback period' analysis method is used in this respect, as it draws particular attention to the importance of the life-span of the system. Payback period is the time required for the cumulative fuel savings to equal the initial investment (Simonson, 1984) and is often used in deciding whether to adopt a particular system.

Whilst the enthusiasm of some individuals or organisations for energy conservation may be a driving factor which sometimes overrides cost-effectiveness, the major decision making factor for an average consumer is the cost effectiveness and not the quantity of energy saved by the system. In chapter 6 it was shown, with the aid of computer simulations, that energy and cost could be saved annually for a typical sports centre building heated partly by warm-air heating and partly by water-filled radiators. It was assumed that a gas-fuelled boiler was in operation, and that the solar collector system was present to contribute to the energy requirement.

Investors in solar heating systems in higher latitudes such as the UK pay a relatively higher cost on solar heating systems due to the relatively low solar energy gains. Thus, the investors of solar heating systems in the UK save relatively less in regards to their fuel cost. It usually means that longer payback periods are required. A solar heating system at higher latitudes often cannot meet all of the heating load, a conventional back-up system is usually required. The cost of the solar installation is therefore additional to that of the conventional thermal system, such as a boiler. So, the potential energy savings of a solar heating system is very important, since it determines that time-span in which the system can pay for itself.
In this chapter, the cost-effectiveness of the proposed solar collector system is examined in terms of its 'real' payback period (that is, accounting for the effect of interest and inflation rates). The same cost-effectiveness estimation was performed on two solar energy installations in the USA; these have similarities to the system proposed in this work. In one of the installations, a perforated metal wall cladding is used to pre-heat the fresh air supply to a factor (SEIA, 1995b).

### 7.2. Analysis technique

To estimate the real payback period of an installation, the standard technique of Net Present Value (NPV) was used (Helcké, 1981). NPV is defined as difference between present value (PV), the aggregate cost savings due to its operation, and the capital cost. In order to use the technique, there are 6 factors which need to be considered. They are the interest rate for an investment, the inflation rate (for fuel), capital, labour cost, maintenance costs, and the net cost savings due to the solar collector operation.

Equations used for the analysis (Helcké, 1981) are as follows. The present value is defined as (Equation 7.1):

$$PV = \sum_{n=1}^{N} \frac{S_n}{(1 + i)^n}$$

...(7.1)

where

- \(i\) = interest rate (-)
- \(n\) = \(n^{th}\) year after investment (-)
- \(N\) = number of years (-)
- \(PV\) = Present Value (£)
- \(S_n\) = amount of the annual energy saving expected after \(n^{th}\) year (£)

It is appropriate to compare the present value \((PV)\) of an investment's total earnings with its capital cost \((C)\). The difference between the present value and the capital cost is called the net present value \((NPV)\):

$$NPV = -C + \sum_{n=1}^{N} \frac{S_n}{(1 + i)^n}$$

...(7.2)
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where

\[ C = \text{capital cost (\pounds)} \]
\[ NPV = \text{Net Present Value (\pounds)} \]

If the effects of inflation rate (\( r \)) is included in this mechanism, the present value (\( PV \)) then becomes:

\[ PV = \sum_{n=1}^{N} S \left( \frac{1+r}{1+i} \right)^n \]  

...(7.3)

and Equation 7.2 becomes,

\[ NPV = -C + \sum_{n=1}^{N} S \left( \frac{1+r}{1+i} \right)^n \]  

...(7.4)

where

\[ r = \text{inflation rate (-)} \]
\[ S = \text{amount of the annual energy savings expected at commencement of project (\pounds)} \]

Equation 7.4 was then used for the analysis.

When \( NPV \) value becomes positive, the system has paid for itself completely, as can be understood from the definition of 'real' payback period (see earlier).

7.3. Cost effectiveness of the proposed system

In order to estimate the NPV, it is necessary to insert appropriate values in Equation 7.4. Inflation and interest rates fluctuate annually. However, it is possible to make estimates based on the previous long-term average inflation and interest rates. Based on the figures in the past 8 years in inflation rates (Norwich Union, 1997), it was assumed that an inflation rate of 4% would apply. For the interest rate, an assumption of 8% was made. Analysis carried out in chapter 6 showed that the net annual cost savings of £694 would be made if gas is used as the fuel for heating, and £1644 if electricity is used. The cost savings on fuel in each year were referenced back to the year 1997 using the estimated inflation and interest rates. Table 7.1 presents a list of items of equipment necessary for retrofitting the solar collector system,
together with their materials and labour costs for the year 1997 (David Langdon & Everest, 1997). From Table 7.1, the polycarbonate cover system, including the associated ductwork and control devices brings the cost of the system to an estimated £111101. The maintenance cost was assumed to be £300 which would be required to be spent once every 15 years. The maintenance cost is also subjected to the inflation rate. It is based on the frequency of maintenance for a profile steel surface finishing in temperate climatic zone (British Steel, 1988).

<table>
<thead>
<tr>
<th>Items</th>
<th>Material + labour rate (£)</th>
<th>Quantity required</th>
<th>Total cost required on item (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triple-layered polycarbonate sheet **</td>
<td>20.28</td>
<td>276.2 m²</td>
<td>5601.34</td>
</tr>
<tr>
<td>90° rect. radius bend (20mm by 800mm)*</td>
<td>33.16</td>
<td>20 no.</td>
<td>663.20</td>
</tr>
<tr>
<td>Dampers for circular section of dimensions 1000mm, electro-magnetic shutter release*</td>
<td>266.77</td>
<td>1 no.</td>
<td>266.77</td>
</tr>
<tr>
<td>Electrical thermostat; installed and connected to provide system control (8 amp; metal base; plastic cover)*</td>
<td>79.16</td>
<td>1 no.</td>
<td>79.16</td>
</tr>
<tr>
<td>Sensors for electronic thermostats (-20 to 70°C nickel plated copper tube)*</td>
<td>20.73</td>
<td>1 no.</td>
<td>20.73</td>
</tr>
<tr>
<td>Circular duct (1000 mm)*</td>
<td>139.68</td>
<td>32 m</td>
<td>4469.76</td>
</tr>
<tr>
<td><strong>Total cost:</strong></td>
<td><strong>11100.96</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1: Material and labour cost of items required for the proposed solar collector retrofit installation (*David Langdon & Everest, 1997; **Wickes Home Improvement Centres, 1998).

The NPV was then calculated using Equation 7.4 and the results are shown in Table 7.2 and 7.3 for use of gas and electricity, respectively, as fuels.

Table 7.2 presents the yearly NPV of the solar collection installation when gas is used as the fuel for heating the sports centre building. A cost of 1.375 pkWh⁻¹ for gas (Roberts, 1988) has been assumed, and the roof-mounted system of dimensions 8.63 m long by 32 m wide has been considered. It can be seen that the value of NPV becomes positive after 26 years. This figure corresponds to the 'real' payback period without taking maintenance cost into account. When the maintenance cost is taken into account, a 32 year 'real' payback period was estimated. When the analysis is repeated with electricity as the fuel, at a cost of 5 pkWh⁻¹ (Gill, 1998), the 'real' payback period reduces to 9 years (see Table 7.3).
### Table 7.2: NPV of the proposed solar collector when gas is used

<table>
<thead>
<tr>
<th>Year number (yr)</th>
<th>Present value (£)</th>
<th>NPV (excluding maintenance cost) (£)</th>
<th>Maintenance cost (accounted for inflation rate) (£)</th>
<th>NPV (including maintenance cost) (£)</th>
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</thead>
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<td>-8572.98</td>
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<td>0</td>
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<td>10176.87</td>
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<td>23</td>
<td>10468.15</td>
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<td>11278.87</td>
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<td>901.85</td>
<td>0</td>
<td>361.57</td>
</tr>
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<td>30</td>
<td>12226.47</td>
<td>1125.51</td>
<td>973.02</td>
<td>-387.79</td>
</tr>
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<td>31</td>
<td>12441.85</td>
<td>1340.89</td>
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<td>32</td>
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<td>1548.29</td>
<td>0</td>
<td>34.99</td>
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</table>
Chapter 7 - Economic assessment of the proposed solar collector system

<table>
<thead>
<tr>
<th>Year number (yr)</th>
<th>Present value (E)</th>
<th>NPV (excluding maintenance cost) (E)</th>
<th>Maintenance cost (accounted for inflation rate) (E)</th>
<th>NPV (including maintenance cost) (E)</th>
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<td>18674.75</td>
<td>0</td>
<td>18674.75</td>
</tr>
</tbody>
</table>

Table 7.3: NPV of the proposed solar collector when electricity is used

7.4. Comparison with other systems and discussion

In Canada, a system called 'Solarwall' has recently been developed (Carpenter and Kokko, 1991). The system has three versions. The first version is a glazed version. The second version is a perforated-plate version. The third version is an unglazed (bare plate) system. They have been designed to be a pre-heater for ventilation air to a building, and thus have some similarities to the system proposed in the current study. For the glazed version, it is comprised of a transparent fibreglass-reinforced plastic glazing which covers a profiled metal cladding to form a duct. The profiled cladding itself is attached to an existing wall of the building. For the perforated-plate version, it is comprised of a perforated opaque metal screen which overclads...
an existing wall of a building to form a duct; air is drawn through the perforations, removes heat from the screen, and enters the building via the duct. In this way, pre-heated ventilation air is supplied to the building. For the unglazed version, it is comprised of a profiled metal absorber which overlads an existing wall of a building to form a duct; air is drawn through the opening at the bottom of the collector, removes heat from the profiled metal absorber, and enters the building via the duct. These arrangements have some similarities to the system proposed in the current study.

The glazed version of 'Solarwall' system, has been installed in the USA at two plants belonging to the Ford Motor Company. At the plant in Chicago, Illinois, a wall area of 5388 m² has been clad with 'Solarwall'; at the other plant in Buffalo, New York, the area of 'Solarwall' is 4645 m². In both cases, large quantities of ventilation air are pre-heated before entry into the factory. The air flows through these collector systems are 130.26 kgs⁻¹ and 195.22 kgs⁻¹, respectively. These figures compare with a figure of 2.51 kgs⁻¹ (total mass flow rate 8.44 kgs⁻¹ for whole building) for the UK sports centre building used as an example case study in this study.

Despite the fact that the two factory installations in the USA are significantly different from a UK sports centre application (in terms of location, scale and ventilation need), nevertheless it is interesting to make comparisons in terms of energy performance, payback period and likelihood of investment. Both factories use natural gas as the fuel, as might the sports centre. Data about the US installations was obtained (SEIA, 1995a and 1995b). This data is presented in Table 7.4, together with the corresponding figures calculated for the UK sports centre installation (the 'proposed' installation) for both gas and electricity as fuels. Note that in all cases it was necessary to assess both simple and 'real' payback periods. The UK sports centre installation used in this comparison was for the case of the solar collector system of area 276.2 m² (8.63 m long by 32 m wide) attached to the roof (see section 6.6 in chapter 6).

For the estimation of the 'simple' payback period and the 'real' payback period of the US installations, inflation and interest rates of 4% and 8% were assumed (US Senate Joint Economic Committee, 1998); they were based on the figures for the past 10 years.
Chapter 7 - Economic assessment of the proposed solar collector system

<table>
<thead>
<tr>
<th>Collector area (m²)</th>
<th>Total energy needed annually for application (MWh)</th>
<th>Total annual energy requirement supplied by solar (MWh)</th>
<th>Percentage of annual load met by solar (%)</th>
<th>Total annual energy requirement met by backup system (MWh)</th>
<th>Solar system capital cost (£)</th>
<th>Capital cost per metre square (£m⁻²)</th>
<th>Fuel price (£.MWh⁻¹)*</th>
<th>Annual cost savings (£)</th>
<th>Annual cost savings per unit collector area (£m⁻²)</th>
<th>Simple Payback (yr)</th>
<th>'Real' payback period (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor Company, Chicago</td>
<td>5388.37</td>
<td>124674.24</td>
<td>18417.12</td>
<td>14.77</td>
<td>106257.12</td>
<td>507444.00</td>
<td>94.17</td>
<td>2.67</td>
<td>49132.38</td>
<td>9.12</td>
<td>10.33</td>
</tr>
<tr>
<td>Ford Motor Company, Buffalo</td>
<td>4645.15</td>
<td>109946.00</td>
<td>9340.32</td>
<td>8.50</td>
<td>100605.68</td>
<td>376320.00</td>
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<td>2.67</td>
<td>24917.69</td>
<td>5.36</td>
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<td>Proposed installation (gas)</td>
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<td>46.93</td>
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<td>11100.96</td>
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<td>13.75</td>
<td>645.27</td>
<td>4.38</td>
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</tr>
<tr>
<td>Proposed installation (electricity)</td>
<td>147.20</td>
<td>457.60</td>
<td>46.93</td>
<td>10.26</td>
<td>410.67</td>
<td>11100.96</td>
<td>75.41</td>
<td>50.00</td>
<td>2346.45</td>
<td>15.94</td>
<td>4.73</td>
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</table>

Table 7.4: Comparison of performances and payback periods between 'Solarwall' installations in USA (SEIA, 1995a,b) and the proposed solar collector system in UK. (Note that * represents payback period accounting for maintenance cost). UK-US exchange rate was assumed to be 0.68 £$/1.

It is interesting to inspect the figure in Table 7.4. Although there are significant differences between the scales of the installations in the US and the UK, the percentage of the annual loads met by solar energy are comparable. Using the figures in Table 7.4, it is possible to estimate the annual solar energy supplied per unit area of collector for each installation; these were found to be: 3.418, 2.011 and 0.319 Whm⁻² of collector Chicago, Buffalo and UK (Kew, London), respectively. However, this cannot be taken as a comparison of collector efficiencies, since the results are affected by other factors such as local irradiance, wind speeds, temperatures and facade orientations.

Table 7.5 gives the comparison of weather conditions between London (UK), Buffalo and Chicago. The differences in average annual sunshine duration go some way towards explaining the above performances, but other factors are involved.
Chapter 7 - Economic assessment of the proposed solar collector system

Table 7.5: Comparison of weather conditions between London (UK), Buffalo and Chicago (US) (CIBSE, 1986; *ASHRAE, 1993). + Buffalo’s sunshine duration was taken as that for Toronto*.

<table>
<thead>
<tr>
<th>Location</th>
<th>Design temperature Summer</th>
<th>Precipitation (mm)</th>
<th>Sunshine</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Average Monthly</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>diurnal range</td>
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<tr>
<td></td>
<td>(°C)</td>
<td></td>
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<tr>
<td></td>
<td>Winter</td>
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</tr>
<tr>
<td></td>
<td>(°C)</td>
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</tr>
<tr>
<td></td>
<td>Wettest month</td>
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<tr>
<td></td>
<td>(°C)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Driest month</td>
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<tr>
<td></td>
<td>(°C)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Annual average duration</td>
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<td></td>
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<tr>
<td></td>
<td>(hr)</td>
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<td>51.48N 0.45W 25 July 29 9 20</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td>617</td>
<td>1475</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>2029+</td>
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</tr>
<tr>
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<td>24</td>
<td>-20</td>
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<tr>
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<td>843</td>
<td>2565</td>
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With regard to the economic performance, Table 7.4 shows that the capital costs per unit area of collector are comparable for both systems ('Solarwall' and the proposed transparent-covered profile cladding), with the proposed UK system being perhaps slightly less expensive. The large difference in the fuel prices between the US and the UK (possibly due to taxation and/or private agreement between supplier and user in this case) makes direct comparison very difficult. However, it is possible to estimate the payback periods and to view these within individual national contexts. For the UK, a simple payback period of 17 years was estimated for the proposed system as part of a gas-fueled sports centre; consideration of UK interest and inflation rates yields a corresponding 'real' payback period of 26 years. For an electrically-fuelled installation, however, the corresponding 'simple' and 'real' payback periods become 5 and 9 years, respectively. In the UK, a payback period of up to 5 years is considered acceptable for thermal equipment such as solar collectors (Moss, 1997), with a period of up to 10 years being considered acceptable in 1988 (Evan, 1988). Thus, retro-fitting of the proposed collector system to a typical sports centre which is gas-fueled is presently not cost-effective. In addition, the payback period is of the same order of magnitude as the life-expectancy of such a building. However, for a similar installation which is electrically-fuelled, such an investment does begin to approach cost-effectiveness. It should be noted that the case study presented (a sports centre) represents a relatively conservative example; buildings that require large quantities of ventilation fresh air may well prove to be a more cost-effective application of the proposed retrofit collector technology. The collector model presented in Chapter 2, together with the simulation tool presented in chapter 6 can be used to evaluate the energy performance of such a building, as well as other building types with standard backing.
insulation. Cost-effectiveness can be estimated using the standard technique employed in this chapter.

In the USA, the situation appears to differ somewhat. A simple payback period up to 15 years, and a 'real' payback period (considering US interest and inflation rates) of up to 24 years appear to be acceptable (as estimated in Table 7.4). This might suggest that US investors are prepared to take a longer-term view when considering energy-saving technology. It is possible that certain UK investors might take a similar view, especially if other benefits (such as good publicity from demonstration of a clean, renewable energy technology) are likely. The present concerns about global warming and the need to meet the national levels of carbon dioxide emissions as agreed at the Kyoto summit in 1997 (Weston, 1997) may thus overcome reservations about current cost-effectiveness of the systems discussed. Note that the payback period estimated here from information supplied for the Chicago installation differs significantly from that of 2 years, as reported by Røstvik (1992) and SEIA (1995) (see Chapter 1). This is thought to be caused by the difference assumptions on interest and / or inflation rates used in the analyses in this study and that reported.

The proposed solar collector system can also be used in a number of ways including, for example, being integrated into an air-source heat-pump system. A combined system such as this is referred to as a solar-assisted heat pump system (Loveday, 1992). Such a system would not only reduce the operation of an existing heating system (thus giving more energy saving for heating purposes), but also it can make use of the heat captured by the collector during the summer to heat water. The payback period of such a system would be dependent upon the cost of the heat pump, as well as ancillary equipment. The cost effectiveness of such a system can be evaluated using a similar approach as used in this study, with the aid of the solar collector model presented in chapter 2, the simulation tool used in chapter 6, and a heat pump model already included in the component library of the TRNSYS simulation program.

7.5. Summary

In this chapter, the technique of net present value has been used to estimate the likely cost-effectiveness of the proposed solar collector system, based on
the case study of a retrofit to a typical UK sports centre. It was found that a 'real' payback period of 26 years (not including maintenance) applied for the case of a gas-fuelled installation, with a corresponding period of 9 years for an electrically-fuelled installation. The former was considered to be unattractive to a UK investor, with the latter just approaching cost-effectiveness. Comparison has been made with two operational installations in the USA, each using 'Solarwall', a recently-developed system that has some similarity to the one proposed in this research. From published figures, the 'real' payback period for the US installations has been estimated to be up to 24 years, suggesting that some US investors might be prepared to take a longer term view of investment in renewable energy technology. Other current factors, such as limiting carbon dioxide emissions as agreed at the recent Kyoto summit, may lead to a change in attitude in the UK towards such investments. It should be remembered that the case study adopted, that of a sports centre retrofit, could be regarded as a conservative application for the proposed solar collection technique; buildings with greater requirements for ventilation fresh air might prove to be a more cost-effective application. In addition, the collector system can be integrated with other building services systems, such as a heat pump, for example. The evaluation of such other systems can be carried out using the building-integrated collector model presented in Chapter 2, together with the enhanced TRNSYS simulation program presented in Chapter 6; cost-effectiveness can then be estimated using the standard technique demonstrated in Chapter 7.
References

ASHRAE (1993), 'ASHRAE Handbook Fundamentals', ASHRAE, Atlanta


Evan B. (1988), 'Active and passive lab design', Architects' Journal, 204, 3 October

Gill R. (1998), Private Communication, 15th February


Norwich Union (1997), 'Norwich Union - UK Inflation Report', Norwich Union Investment Economics Department, September
(URL: http://www.norwich-union.co.uk/economic/ecukpi.htm)

Roberts G. (1998), Private Communication, 10th February


Wickes Home Improvement Centres (1998), Product catalogue for June to August
Chapter 8 - Final Summary, Conclusions and Recommendations

8.1. Final Summary

This thesis presents a study of the potential of covered profiled steel cladding as a building-integrated solar collector in the UK climate. The study began with a literature survey of existing collector systems and existing mathematical models of collector thermal performances. From this, the novelty of the proposed collector concept and system was established, despite the fact that some collector design had similarities to that proposed. One of the assumptions used in existing mathematical models of collector performance is that front and rear ambient temperatures \((t_a \text{ and } t_a')\), respectively, are equal; this leads to the traditional performance estimation of collectors in terms of the heat removal factor \((F_R)\) and the overall heat loss coefficient \((U_L)\) as derived from the standard Hottel-Whillier-Bliss analysis. This assumption may be inappropriate for some building-integrated collectors, and thus a steady state mathematical model was derived which allowed for differing front and rear ambient temperatures, the inclusion of the long-wave radiation heat transfer mechanism between absorber and sky, and absorber area enhancement as a result of the profile geometry. One important contribution of this work is the introduction of an equivalent ambient temperature, \(t_{a'}\); this allows the thermal performance of a structurally-integrated solar collector (where \(t_a \neq t_a'\)) to be compared with that of a collector with front and rear ambient temperatures being equal, and for the comparison to be made in terms of the conventionally-accepted terms of \(F_R\) and \(U_L\). Using the modelling approach presented, correlations have also been derived which permit calculation of collector surface temperatures.

The proposed model, which represents an extension to the existing Hottel-Whillier-Bliss analysis, was further tested for conditions when \(t_a \neq t_a'\); and for a range of rear insulation levels. It was found that for the conditions when the rear conductance exceeds about 2 Wm\(^{-2}\)K\(^{-1}\), and the front / rear ambient temperature difference exceeds about 15 °C, predictions of collector performance using the model proposed begin to differ significantly from those obtained using the standard HWB analysis. Note that use of the latter required the introduction of a weighted ambient temperature \(t_{a''}\).
Laboratory measurements of the thermal performance of a test section of the proposed solar collector were carried out, allowing performance parameters to be obtained for design purposes. A test rig was designed for this purpose, as well as that of mathematical model validation for this application. Published Standards for the thermal performance measurement of solar collectors (ASHRAE, 1986; British Standards Institution, 1986) were used only as references against which the test rig performance could be compared, the rig not being set up to be standard test facility. The optical properties of the cover and absorber were measured so as to determine the value for \((\tau \alpha)_c\) to be used for both calculation of \(U_L\) and \(F_R\) from laboratory measurements and their prediction from the mathematical model. Measured thermal performances showed satisfactory repeatability; however, an axial heat conduction mechanism was identified for the test section. The magnitude of the heat conducted was estimated and was used to 'revise' the predicted values of \(U_L\) and \(F_R\). When these additional heat losses (due to axial conduction) were taken into account in the predicted thermal performances using both the derived model and an existing model (Duffie and Beckman, 1991), satisfactory agreement was observed between the laboratory measurements and the predictions of the derived mathematical model. Further suitability of the derived model for the proposed application was also confirmed by the reasonably good agreement between predicted and measured collector surface temperatures. The validation was carried out for a laboratory situation where front and rear ambient temperatures were equal (a 'special case' of the more general situation where \(t_a = t_r\)), and for the standard level of backing insulation as found for the proposed application. This confirmed the models suitability for this application, and that the standard HWB analysis is suitable for this situation. However, the range of conditions are also identified for front / rear temperature differences and levels of rear insulation for which the standard Hottel-Whillier-Bliss analysis is inappropriate, and for which the proposed model is needed.

The next stage of the research was to evaluate the energy performance of a typical building in the UK climate that was equipped with the proposed solar collector system. Retail outlets, supermarkets, offices and sports centres were among the buildings considered, since these make extensive use of profiled metal cladding as part of their external envelope. A sports centre building was selected as the case study, based on the likelihood of their owners investing in a (renewable) energy-saving system. Furthermore, such a case study was
considered to be a 'conservative' and typical test of the proposed system; while a building with a large outdoor air requirement may be a more likely 'beneficiary' from such a system, there are likely to be less of these in the UK building stock. However, the collector system proposed could be utilised with any of the building types described, provided that there is a fresh air requirement; the proposed system can also be used in conjunction with a range of heating systems, not only the warm-air system considered in the case study. A literature review of sports centre design was then undertaken in order to produce the typical case study example for this building type (Chapter 5).

The mathematical model of thermal performance for the structurally-integrated collector (derived in Chapter 2) was then encoded as a new model in the TRNSYS simulation program component library. In order to simulate the annual energy performance of the sports centre case study building, it was necessary to 'integrate' the collector system within the simulation environment, as well as to introduce a file of standard UK weather data to TRNSYS. Once this had been achieved, simulations were performed, firstly of the sports centre without the solar collector system; the energy performance figures obtained from simulation were compared with published results, and were found to be generally in agreement. This was considered to be a validation of the sports centre simulation model, which then became the 'base case' against which the solar-assisted versions of the sports centre could be compared. A parametric study was then conducted for the solar collector facade facing five orientations, and attached at two positions (wall and inclined roof); collector areas, aspect ratios, air gaps widths and control strategies were also varied in the simulations. In this way, it was possible to formulate general design guidance for maximising the energy performance of the collector system (Chapter 6).

The cost-effectiveness of the proposed solar collector system was then evaluated using the technique of 'net present value'. This took account of material, labour and maintenance costs, as well as (estimated) interest and inflation rates. The 'real' payback period (without maintenance costs) was estimated to be 26 years for a gas-fuelled installation. These results were found to be comparable to those for two operational installations in the USA, the latter employing 'Solarwall' - a system having some similarities to that proposed (Chapter 7).
The contributions to knowledge offered by this research may therefore be summarised as follows:

i) a mathematical model of the thermal performance of a novel air-heating solar collector geometry in which the front and rear ambient temperatures can be allowed to differ;

ii) the introduction of the equivalent ambient temperature \( (t_a') \) concept, offering the ability to compare the performance of such a collector (in which \( t_a \neq t_a' \)) with a collector which has been analysed in the conventional way, that is, front and rear ambient temperatures are equal \( (t_a = t_a') \);

iii) the identification of those conditions of front / rear ambient temperature difference and rear insulation levels for which the standard Hottel-Whillier-Bliss analysis is adequate, together with the conditions requiring treatment with the proposed model;

iv) a set of measured performance characteristics for the proposed collector geometry, which are suitable for use by designers;

v) the creation of a new collector component model within the TRNSYS library, thereby allowing the performance to be simulated of a range of buildings which might incorporate such a structurally-integrated collector; with appropriate treatment of internal heat generations, the modelling can be extended to include ventilated photovoltaic facades;

vi) general design guidance for maximising the energy performance of such a collector system, when attached to a typical profiled-clad building in the UK, such as a sports centre;

vii) an evaluation of the cost-effectiveness of carrying out an option such as in vi), enabling an investment decision to be made; payback periods are estimated.
8.2. Conclusions

i) A mathematical model has been derived of the thermal performance of a building-integrated air-heating solar collector, in which front and rear ambient temperatures can be allowed to differ. This was shown to give significant differences in the prediction of $U_L$ and $F_R$ in comparison with that of the standard Hottel-Whillier-Bliss analysis for the conditions where rear insulation is less than about 2 Wm$^{-2}$C$^{-1}$ and where the front / rear temperature difference exceeds about 17°C. The concept of the equivalent ambient temperature $t_a^*$ has been introduced and defined, and provides a means for determining values for $U_L$ and $F_R$ for the situation when front and rear ambient temperatures differ.

ii) The attachment of a transparent cover to a profiled steel building facade can produce an air-heating solar collector, the thermal performance of which can be modelled for differing front and rear ambient temperatures. In terms of the conventionally-accepted parameters which are used to describe collector performance, covered profiled steel cladding has the following properties:

- heat removal factor, $F_R$: 0.7 ± 0.2
- overall heat loss coefficient, $U_L$: 11.8 ± 2.0 Wm$^{-2}$C$^{-1}$
- $(\tau\alpha)_C$ for triple-layered polycarbonate cover: 0.68

The above figures can be used by designers to give an estimate of the energy performance of such a system.

iii) The inclusion of the collector model as a new TRNSYS component allows the performance of a range of buildings to be simulated which might utilise a structurally-integrated covered profiled steel collector. For a case study of a retrofit to a typical sports centre in the UK, the following general design guidance is offered:

- Use a facade which faces as close to south as possible.
- Depending on local surroundings, attach the collector system to the roof, rather than to the vertical facade; (this may also be preferable
from the points of view of vandalism risk and glare to passers-by, motorists, etc.).

- The area of the collector system to install will depend upon the utilisation (Simonson, 1984) of the collected energy by the building (i.e. appropriately match the building load to the supply).

- Heat removal efficiency from the collector can be maximised by increasing the airspeed through the collector flow channel. The latter can be achieved by reducing the gap between profiled steel surface and the underside of the transparent cover. Additionally (or alternatively), for a given area, the aspect ratio should be selected to maximise the length of air flow. Note, however, that fan power requirements should simultaneously be considered (net cost benefit). The optimum collector channel depth has been identified here as 0.02m. Note that this channel depth was estimated for a collector dimensioned 8.3m long and by 32m wide and for a flow rate of 2.51 kgs⁻¹.

- The setting of the irradiance level threshold for the control of the current collector system was 0 Wm⁻². This threshold setting disables the collector operation when the irradiance level on the collector surface reduces to 0 Wm⁻². In the simulation exercise, this setting gave the best cost saving compared with higher threshold settings. Concerning the outdoor air temperature threshold set-point, a highest possible set-point should be used, provided the temperature of the zone (to which the solar pre-heated fresh air is delivered) can be regulated (by opening of windows, natural ventilation, etc.) to a reasonable level without causing thermal discomfort.

- It will be necessary to consider the inclusion of some form of flow channel venting system, so as to prevent excessive collector surface temperatures during periods of high solar irradiance.

- The simulation program, together with its new ‘facade-integrated collector’ component model, should be used to compare alternative design solutions in more detail, on an individual case basis. The cost-effectiveness of different solutions can then be subsequently evaluated.

iv) For the case study of the UK sports centre solar collector retrofit, a cost-effectiveness evaluation gave a 'real' payback period of 26 years for a
gas-fuelled installation. Whilst investment in the solar collector system is approaching cost-effectiveness for an electric installation, it is currently not a cost-effective option for a gas-fuelled installation, especially since the 26 year payback is also of the order of the lifespan of such a building. It may thus be more cost-effective to consider employing such a collector system on a building with a large fresh air requirement (high occupancy, such as auditoria, for example), and to displace electricity as the main fuel; the cost-effectiveness of such an installation should be confirmed via simulation.

v) The payback periods in iv) are of a similar order of magnitude to those estimated for two operational installations in the USA which use 'Solarwall' - a collector system having some similarities to that proposed in this research. Whilst this may suggest a different attitude in the USA towards investment in energy-saving technology compared to that in the UK, it may also indicate that there are other benefits that are worth considering; examples of these are technology demonstration and good publicity for the building owner concerned by being seen to promote 'green' solutions. As the world moves into the next millennium, the need for renewable energy solutions may well start to influence decisions to a greater extent than that of simply cost. The collector system presented here may then be adopted on a large scale, especially in view of its simple retrofit capability.

8.3. Recommendations

Ideas for improvements have arisen at different stages while carrying out the research. These ideas has given rise to recommendations for further work. The following are the recommendations.

8.3.1. Mathematical modelling

The mathematical model derived has been validated for use in the proposed application, that is, for a standard level of rear insulation as found in profile-clad buildings, and for which the standard Hottel-Whillier-Bliss analysis has been shown to be adequate. In addition, those conditions of front / rear ambient temperature difference and levels of rear insulation have been identified for which analysis using the proposed mathematical model is required. It is recommended that validation of the model is carried out for
these latter conditions of use. This could be done by constructing a box through which heated air can be passed, and attaching this to the rear of the test collector of interest (e.g. one with little or no rear insulation). In this way, different rear temperatures can be set up, and collector performance measured, for comparison with model predictions.

It was found by computer simulation (Chapter 6) that the surfaces of both polycarbonate cover and profiled steel cladding in the solar collector may reach unacceptably high temperatures at certain times when operated in the UK climate. This could happen during the summer period when the solar irradiance is high and the solar collector is not being operated for space heating. It was suggested that the problem may be resolved by providing openings at the collector outlet so as to encourage natural ventilation through the solar collector and hence discharge the heat to the outside environment. To model this situation, a correlation that describes the natural convection process in the collector flow channel needs to be incorporated into the proposed model. In this way, the effect on the surface temperatures attained may be predicted.

8.3.2. Solar simulator

The solar simulator for this study consisted of 4 CSI 1000W lamps. Although the irradiance distribution produced was considered to be satisfactory for validation purposes, the irradiance distribution would be considerably improved by increasing the number of lamps from 4 to 7, with one lamp placed at the middle and behind the hexagonal geometry formed by the other lamps (Figure 8.1). For a solar simulator arrangement consisting of 7 Compact Iodide Daylight (CID) lamps, the distance between the lamp at the middle and the hexagonal geometry formed by the other 6 lamps should be 500mm. For the same solar simulator geometry consisting of CSI lamps, this distance would be different. Improvement to the irradiance distribution of the existing CSI lamps can be made by simply adding an extra CSI lamp at the middle and behind the square array formed by the existing CSI lamps. As the irradiance from each lamp to a surface can be estimated from the correlation derived by curve fitting to reported data (Krusi and Schmid, 1983), the distance behind at which the extra lamp should be located may be estimated, so as to improve the irradiance distribution nearer to that required by the standards (ASHRAE, 1986; British Standards Institution, 1986).
Figure 8.2 shows the spectral distribution of CSI and CID lamps along with the direct normal solar spectral irradiance for 1.5 air mass (PROBE, 1995). As clearly shown, a CID lamp has a spectral distribution more closer to that of the direct normal solar spectral irradiance for 1.5 air mass than a CSI lamp. This suggests that CID lamp is a better lamp type to be used for solar simulation.

Figure 8.1: A 3-dimensional view of a solar simulator geometry consisting of 7 Compact Iodide Daylight (CID) lamps (PROBE, 1995)

Figure 8.2: Spectral distribution of CSI and CID lamps compared with that of natural sunlight (1.5 air mass) (PROBE, 1995)

8.3.3. Axial conduction

The main requirement for validating the thermal model was found to be the need for accurate determination of the energy balance on the laboratory test section. An additional heat loss mechanism was considered to be that of axial conduction. This heat loss was initially not accounted for in the mathematical model. Although this problem was eventually solved and the model validated by accounting for this heat loss in the predicted thermal performance, this problem may be avoided in the future by providing better insulation around the collector and between the test section and the attached ductwork. The former can be achieved by providing better insulation on both edges, as suggested by the ASHRAE Standard (1986); in addition, the depth of the edge plates needs to be reduced to the same depth as that from the absorber plate to the top surface of the polycarbonate cover, so as to reduce heat loss.
from the test section by convection to the simulated wind. The latter can be achieved by inserting a thermal 'break' in the profiled steel cladding at entrance and exit ends to the test section, thus separating the test section part of the absorber from those parts of the absorber used to assist the flow development.

8.3.4. Full validation of the proposed model

It was found in Chapter 4 that the predictions of collector thermal performance using the proposed model begins to differ significantly from those obtained using the standard HWB analysis (with the introduction of $t_a$”) when the rear conductance exceeds about 2 Wm$^{-2}$°C$^{-1}$ and the front / rear ambient temperature difference ($t_a' - t_a$) exceeds about 15°C. Therefore, it is recommended that full validation of the proposed model is carried out over a range of conditions, using a specially constructed solar simulator in which front and rear ambient temperatures can differ.

8.3.5. Computer simulation and cost benefit estimation

Due to the comparatively long payback period estimated for the proposed solar collector system, simulations of alternative collector geometries and/or operational modes are recommended. Examples here might include buildings with large requirements for pre-heated fresh air (auditoria, factories housing certain processes). The TRNSYS program together with its new component model can be used to do this.

It can be seen from Chapter 7 that one of the main contributions to the cost of the proposed solar collector system is the cost of the associated ductwork. To reduce the cost of the proposed collector system, it is possible to eliminate the ductwork and operate the proposed solar collector in the manner reported by MacGregor (1979). The system reported by MacGregor (1979) comprises of transparent insulation as an 'add-on' to an existing masonry wall, creating an unvented cavity between them. This mode of solar utilisation, when the proposed solar collector is to operate in the manner reported by MacGregor (1979), allows the inner layer (profile steel in the case of this study) to be heated by the solar irradiance while the heat loss from the facade is reduced, since the cover system (polycarbonate cover in this case) retains a layer of still air to increase the thermal resistance of the component. Whilst this is
feasible and would be effective in energy saving to a certain extent, the cost benefit and the payback period of the proposed collector, operated in the manner reported by MacGregor (1979), needs to be examined before any conclusion about such a system can be made. It is recommended that such a system as described above should be studied.

Alternatively, a collector system using the proposed solar collector with less ductwork may be used. In this case, the pre-heated fresh air would be fed to the sports hall directly (without going to the AHU) serving in parallel with the existing AHU (Figure 8.1). Though the cost benefit for such an arrangement has not been estimated, such an arrangement could result in a shorter payback period due to the reduced capital expenditure on ductwork. Therefore, it is recommended that the thermal performance of such an arrangement should be estimated using the TRNSYS computer simulation, thus allowing the cost benefit of such an arrangement to be estimated.

Figure 8.1: Alternative arrangement for the proposed solar collector

Another mode of operation could be to use the proposed solar collector as the heat input to an air-source vapour compression cycle heat pump (Rogers and Mayhew, 1978). A domestic heat pump system has been evaluated by Loveday (1992). Evaluation of thermal performance on such a system for a domestic-scale system (with an insulated heat store) indicated a savings of 64% of heating energy is possible. In this way, heat that might otherwise be wasted during summer (when heat is not needed for space heating) could be utilised for hot water heating, with the addition of an appropriate heat recovery heat exchanger.

The thermal performance of such a combination may be estimated by simulation using the mathematical model derived in Chapter 2 (now a
TRNSYS component model) and a model describing the performance of an air-source heat pump (an existing component model in TRNSYS).
References


MacGregor K. (1979), 'Solar Cladding - Insolation plus insulation for solid walls', ISES Silver Jubilee, International Congress of ISES. Joint Meeting with the American Section of ISES May 28-June 1, 1979


Appendices
A1. Publications to date

A1.1. Paper 1

COVERED PROFILED STEEL CLADDING AS A SOLAR COLLECTOR: LABORATORY TESTING USING A SIMULATOR

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ABSTRACT

The efficiency characteristic of covered profiled steel cladding is determined using a laboratory solar affiliation simulator. Values for the heat removal and heat loss factors are found to be 0.52 and 16.0 Wm\(^{-2}\)K\(^{-1}\), respectively, for a 1m\(^2\) section of vertical collector covered with a triple-skin polycarbonate glazing. These results are in reasonable agreement with theoretically-predicted values based upon a simple flat-plate air heating collector with flow above the absorber; however, it is recommended that the effects of the trapezoidal absorber profile be included in future modelling.

KEYWORDS

Profiled steel cladding; air-heating solar collector; efficiency characteristic; solar simulator.

INTRODUCTION

Profiled steel is widely used as an inexpensive cladding for many commercial and industrial buildings. Its performance as an integral part of several solar collector designs has already received attention (Loveday, 1991; Wright et al, 1979). It is possible to further utilise the cladding as an air-heating solar collector by covering its surface with a suitable transparent material. By leaving a gap between the profiled steel (which acts as the absorber) and the transparent material (which acts as the cover) an air flow channel above the absorber is created, which offers an enhanced heat transfer area due to the trapezoidal profile. Other advantages of such a collector system are as follows.

i) A major retrofit opportunity exists, due to the already large number of buildings which utilise this type of cladding. In the European Community, over 100 million square metres of profiled cladding are erected annually.

ii) The building envelope would perform the dual function of weatherproof membrane and energy collector, thereby enhancing the cost-effectiveness of construction. Fabric heat loss from the building can be recycled, at least in part.

iii) The attachment of an aesthetically-pleasing glazed cover to the profiled facade is a relatively straightforward procedure, requiring the minimum of structural alteration. At night, the arrangement would provide a reduced U-value for the facade.

Once external air has been pre-heated by such a collector system, it is introduced to the building and can be used either directly, or via a suitable space heating system. Carpenter and Kokko (1991) tested, under Canadian conditions, a 237m\(^2\) collector system comprised of a corrugated absorber with a cover either of...
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fibreglass or of perforated aluminium. In addition to recovering building heat loss, the system benefitted
from the utilisation of ground-warmed air, and a payback of just over two years was reported.

In order to fully evaluate the performance of such a system on a typical UK building, it is necessary to
determine its thermal performance and efficiency characteristic; this paper reports the results of tests on an
initial collector configuration. The specific objectives are:

i) to measure the efficiency characteristic of a vertical section of covered profiled steel;

ii) to compare experimental results with predictions from an existing thermal model, and to comment
on the findings.

TEST UNIT AND SIMULATOR

The collector test unit consisted of a section of profiled steel cladding (British Steel 'long rib/1000W')
painted with a black Pvf2 coating capable of withstanding a temperature of 120°C. The section was 1m
wide and 1.8m long, and was covered with a triple-wall polycarbonate glazing ('Corotherm') of total
thickness 0.016m. A flow channel of width 0.1m was created, measured from the top of the absorber
profile to the underside of the cover; the top to bottom profile depth is 0.03m. The unit was tested in the
vertical position to simulate a wall, and a 0.1m thick rockwool backing was fixed to the rear side of the
profile, similar to that found in conventional construction.

Air was ducted into and out of the collector unit via plenums fitted at the top and bottom, while a fan
simulated a wind of average speed 3.4 ms⁻¹ over the surface of the cover. Four Thorn CSI 1000 Watt
lamps produced a solar irradiance of 312 Wm⁻² over the collector area of 1m² from a distance of 4 metres.
The collector area actually exposed was 1 metre square, though a total length of 1.8m was installed so as
to produce a more representative flow regime as would be found through an element of a larger collector
facade. The surrounding laboratory air temperature was maintained at 18°C ± 1°C, and air entering the
collector was electrically pre-heated, and mixed using a wirewool matrix. Temperature differences
between the inlet and outlet air temperatures, and between inlet and ambient air temperatures, were
measured using calibrated K-type thermopiles.

RESULTS

Figure 1 presents the efficiency characteristic of the collector for an inlet air mass flow rate of 0.169 kgs⁻¹.
Values for the glazing transmission coefficient, τ, and the plate absorption coefficient, α, were estimated
to be 0.7 and 0.9, respectively, giving a value for (τα)α of 0.64 (Duffie and Beckman, 1980). Based on
this figure, the standard Hottel-Whillier-Bliss analysis gives values for the collector heat removal factor,
FR, and overall heat loss factor, U_L, of 0.52 and 16.0 Wm⁻² K⁻¹, respectively. These may be compared
with values for F_q of 0.73 and for U_L of 10.5 Wm⁻² K⁻¹, calculated from expressions reported by Duffie
and Beckman (1980): these expressions describe the performance of a simple flat-plate air heating
collector with flow above the absorber.
Agreement between experiment and calculation is seen to be moderate; it is thought that agreement would be improved by a more accurate determination of cover surface temperatures in the test system, and also by appropriately modifying the Duffie and Beckman model to include, for example, the effect on heat transfer of the trapezoidal profiled absorber.

CONCLUSIONS

The efficiency characteristic of one type of vertical covered profiled steel cladding as a solar collector has been determined experimentally and the plot may be used as a guide to its behaviour, and for design purposes. For analytical studies, the performance can be approximately modelled as that of a simple flat plate collector with flow over the absorber, but it is recommended that the model is modified to account for the effect of profiling, for example. The authors are currently pursuing this approach, together with experimental tests of further collector configurations of this type.

ACKNOWLEDGEMENTS

The technical assistance of M. Barker and D. Sanham in constructing the test system is gratefully acknowledged. The authors thank British Steel and Ariel Plastics Ltd for donating samples for experimentation.

REFERENCES


A1.2. Paper 2

Covered profiled steel cladding as an air heating solar collector: laboratory testing, modelling and validation

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Received 20 November 1996; revised 4 February 1997; accepted 10 February 1997

Abstract

Profiled steel cladding is a widely-used product in construction and can be adapted to act as a building-integrated solar air heating collector. A mathematical model is presented of an air heating solar collector using profiled steel as the absorber and transparent multi-layer polycarbonate rigid sheeting as the cover. The model is validated by experiments conducted within a controlled laboratory environment, and the experimental results are also presented. The expected full-scale performance has been evaluated based on the assumption that the proposed solar collector is to be used as an integral part of a building wall structure situated in an urban area of the UK in winter. The evaluation leads to equivalent values for $U_L$ and for $F_{sh}$ of approximately 4.0 W m$^{-2}$°C$^{-1}$ and 0.71, respectively, which provide an estimation of the performance of such a collector system in the UK.

Keywords: Profiled steel cladding; Solar air heating collector; Mathematical model; Urban area; UK winter

1. Introduction

Corrugated metal has served as a roofing material for barns and houses for a century or more. Today, while corrugated metal remains a common building component, the advent of profiled steel cladding as a more 'up-market' product has led to widespread use of this type of material within the construction industry. Profiled steel cladding is used in many commercial and industrial buildings as an inexpensive means for providing a weatherproof skin in the form of roofs and walls. With the development of more aesthetically-pleasant colours and finishes, and the introduction of a variety of trapezoidal profiles, many types of buildings such as schools, offices, sports halls and hospitals now utilize this building product out of choice rather than cheapness [1].

By making certain adaptations, it is possible to use profiled metal cladding as an air heating solar collector. This can be achieved by attaching a suitable transparent cover material to the external surface of the profiled metal so as to leave a gap between the top surface of the metal and the lower surface of the cover; the profiled metal surface can then act as the absorber plate of the collector. External air is drawn into the collector and flows in the channel formed by the profiled metal and the transparent cover; the air can then remove heat from the absorber for use in the building. The transparent cover could be of glass, but it is quite likely that a polycarbonate material would be employed; a rigid triple-layer type of polycarbonate glazing currently used in conservatories and other structures would appear to be a particularly suitable choice because it can offer some thermal insulation as well as allowing a similar solar transmission to that of glass (manufacturer's data). Fig. 1 illustrates the proposed collector geometry. The advantages of such a system (if shown to be effective) are:

(i) cost-effectiveness, as a result of combining the dual roles of weather-proof skin and energy collector into a single building-integrated component,

(ii) reduction in the effective $U$-value of the building envelope by the addition of the cover layer, and by the potential recycling of fabric heat losses that would otherwise be lost to the external environment,

(iii) the possibility of significant retrofit potential, due to the already widespread use of profiled metal (most often steel) as a building envelope. In the European Community alone, over 100 million square metres of profiled cladding are erected annually.

1.1. Previous studies

Previous studies have reported on the use of profiled metal as a component of an air heating solar collector. Wright et al. [2] investigated the performance of a geometry consisting of air flow beneath the profiled absorber. It was shown that...
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collector consisting of profiled metal as the absorber with air flowing, hem, een absorber and a -lass cover has been reported posed in our study differs in that outdoor air enters the col-

southwest climate of Phoenix, USA, with an air heating solar collector and flows over the absorber. A house in the and

67% could be yielded. Their design used air from inside the cover system and an air flow rate of 0.0 15 m$^3$ s$^{-1}$ per metre

square in the flow channel, a maximum efficiency of about dicted that A-hen using double glazed low iron glass as the car

beneath a corrugated absorber. Experimental testing was carried on a prototype test module under a flow rate of

maximum efficiency was measured to be about 49%. Tbey pre-

Fig 1. (a) Proposed air heating solar collector geometry; (b) geometry of the polycarbonate cover system.

profiled metal is easily installed and functions well as an absorber plate. However, no mathematical model of collector

performance was reported, nor were any performance characteristics measured.

Carpenter and Kokko [3] tested a 237 m$^2$ solar collector in Canada, consisting of a corrugated absorber as the wall of a building with a cover of either fibre-glass reinforced plastic or of perforated aluminium. It was found, for the plastic system, that solar energy could be delivered with approximately 45% efficiency. However, it is necessary to consider more aesthetically-pleasing cover materials than those used in the Canadian study and also to investigate their performance under UK conditions.

Kohler et al. [4] reported on a glazed building-integrated site-fabricated air heating solar collector, with air flow beneath a corrugated absorber. Experimental testing was carried out on a prototype test module under a flow rate of 0.01 m$^3$ s$^{-1}$ per metre square of collector area, and the maximum efficiency was measured to be about 49%. They predicted that when using double glazed low iron glass as the cover system and an air flow rate of 0.015 m$^3$ s$^{-1}$ per metre square in the flow channel, a maximum efficiency of about 67% could be yielded. Their design used air from inside the building as the heat transfer medium. The arrangement proposed in our study differs in that outdoor air enters the collector and flows over the absorber. A house in the arid southwest climate of Phoenix, USA, with an air heating solar collector consisting of profiled metal as the absorber with air flowing between absorber and a glass cover has been reported

[5] with a collector of the same geometrical configuration as that proposed in this paper, but the report failed to provide any information on collector thermal performance or efficiency. Therefore, it seems clear that there is a need for a set of measured performance characteristics for the collector geometry proposed in this study, together with an assessment of energy performance in typical UK conditions.

There are many studies already in the literature that present mathematical models of collector thermal performance. Duffie and Beckman [6] present a summary of models that are available for the most common collector geometries. Choudhury and Garg [7] present a model for a collector with a corrugated absorber and a two-layer cover, a geometry very similar to that proposed. However, they presented a theoretical model only of thermal performance, there being no validation by comparison with measured results. In addition, their model accounted only for the case of air at external ambient temperature being supplied to the collector.

In all cases of collector thermal modelling, the energy balance approach devised by Hottell, Whiller and Bliss [6] is adopted, in which ambient temperatures at the front and rear of the collector are assumed to be equal. If the performance of a profiled cladding collector as an integral part of a building structure is to be properly modelled, then it will be necessary to derive a model which accounts for differing temperatures at the front and rear of the collector, that is, corresponding to the internal and external environment of the building. Furthermore, such a model can then be incorporated within a building thermal simulation program for determining the overall thermal energy performance of the building.

1.2. Objectives

From the preceding discussion, it is evident that there is a need for a set of experimentally-measured performance characteristics for a collector comprised of a trapezoidally-profiled metal absorber, a multi-layer transparent cover, and air flow over the absorber; the experimental results could then be used for estimating the performance of such an arrangement when used as a solar energy collector for a building in the UK. As well as being able to compare directly its performance characteristics with those of other (conventional, purposed-designed) collector types, it will also be necessary to derive a model of collector thermal performance in which front and rear ambient temperatures can differ. The model should also account for the effect of the triple-layer cover and the profiled absorber. This paper presents such a model, validated by laboratory experimentation, together with a set of measured standard performance characteristics. Model predictions are compared with those from an existing model for a collector with a similar flow geometry [6], but with a flat absorber plate and a single-layer cover which is assumed to be infinitely thin and infinitely conducting. A corresponding comparison with the model by Choudhury and Garg [7] was not possible because in their model the inlet air temperature was assumed at all times to be equal to that of the ambient
air. It is necessary to vary the inlet air temperature in a laboratory test in order to generate the collector characteristics.

2. Model derivation

A steady-state approach was used to thermally model the collector geometry illustrated in Fig. 1. This was because this type of collector can be considered as a lightweight component of relatively low mass in comparison with typical masonry walls. Fig. 2 shows the cross-section through the collector, and the heat transfer mechanisms assumed for modelling. The following set of energy balance equations is derived for a unit area of collector.

2.1. Collector energy balance

An energy balance for the solar collector was performed on the cover inner surface, the absorber and the heat transfer fluid, respectively. No solar absorption within the cover system is assumed. The effect of corrugations on the convection and radiation heat transfer is modelled by introducing an area enhancement factor, $K$. This, and all other terms, are defined in the Nomenclature.

\[
\begin{align*}
U_1'(T_2 - t_4) + h_2(T_2 - t_4) &= K h_{22}(T_3 - T_2) \quad (1) \\
U_2'(T_3 - t_4) + K h_2(T_3 - T_2) &= K h_{22}(T_5 - T_3) = l(\tau_0) \quad (2)
\end{align*}
\]

Solving Eqs. (1) to (3) for $T_2$, $T_3$ and $\frac{dT_0}{dy}$ yields:

\[
\begin{align*}
T_2 &= \frac{U_1'}{P_1(U_1' + h_2 + K h_{22})} + \left\{ \frac{h_2}{U_1' + h_2 + K h_{22}} \right\} \left\{ P_1(U_1' + h_2 + K h_{22}) \right\} T_0 \\
&\quad + \left\{ \frac{U_1'}{U_1' + h_2 + K h_{22}} \right\} \left\{ P_2(U_1' + h_2 + K h_{22}) \right\} T_0 \\
&\quad - \frac{1}{P_1(U_1' + h_2 + K h_{22})} l(\tau_0) \quad (4)
\end{align*}
\]

Integrating Eq. (6) and applying the boundary conditions that $t_1 = t_f$ when $y = 0$ and $t_4 = t_4$ when $y = L$ yields:

\[
\begin{align*}
t_0 &= \frac{P_6}{P_3} + t_0 \exp(\tau_3) - P_6 \\
&\quad \times \left\{ \frac{h_2 U_1'}{U_1' + h_2 + K h_{22}} - \frac{h_2 U_1'}{U_1' + h_2 + K h_{22}} \right\} T_0 \\
&\quad + \left\{ h_2 \left( \frac{P_2(U_1' + h_2 + K h_{22})}{U_1' + h_2 + K h_{22}} - \frac{P_2(U_1' + h_2 + K h_{22})}{U_1' + h_2 + K h_{22}} \right) \right\} T_0 \\
&\quad - \frac{1}{P_1(U_1' + h_2 + K h_{22})} l(\tau_0) \quad (6)
\end{align*}
\]

where

\[
\begin{align*}
P_1 &= \frac{K h_{22} - h_3}{U_1' + h_2 + K h_{22}} - h_3 - \frac{1}{U_1' + h_2 + K h_{22}} \\
&\quad \times \left\{ \frac{h_2}{U_1' + h_2 + K h_{22}} \right\} \left\{ P_1(U_1' + h_2 + K h_{22}) \right\} T_0 \\
&\quad + \left\{ \frac{U_1'}{U_1' + h_2 + K h_{22}} \right\} \left\{ P_2(U_1' + h_2 + K h_{22}) \right\} T_0 \\
&\quad - \frac{1}{P_1(U_1' + h_2 + K h_{22})} l(\tau_0)
\end{align*}
\]

The mean fluid temperature can then be derived as:
2.2. Heat transfer coefficients

The following expressions were used in the estimation of the heat transfer coefficients. For the cover outer surface convection heat transfer coefficient, the following expression [8], valid for wind speed less than 5.0 m s\(^{-1}\), was used:

\[ h_1 = 5.7 + 3.8V_w \]  

(11)

The radiation heat transfer coefficient between the outer cover surface and the front ambient environment was evaluated from Ref. [6]:

\[ h_r = \sigma e_r [(T_1 + 273)^2 + (T_2 + 273)^2](T_1 + T_2 + 546) \]  

(12)

The Nusselt number (Nu) for the surfaces forming the air channel was evaluated from Kays' [9] correlation for fully-developed turbulent flow between flat plates with one side heated:

\[ Nu_{A2} = 0.0158 Re_0^{0.8} \]  

(13)

Entrance region effects were accounted using the following correlation [6]:

\[ Nu = Nu_{A2} \left[ 1 + \left( \frac{d_m}{L} \right)^{0.7} \right] \]  

(14)

and

\[ h_2 = h_s + \frac{Nu \cdot k_s}{d_m} \]  

(15)

An expression by Duffie and Beckman [6] was again used to estimate the radiation heat transfer coefficient between the inner cover surface and the absorber surface:

\[ h_{23} = \sigma \left[ \frac{(T_1 + 273)^2 + (T_2 + 273)^2}{(1/e_1) + (1/e_2)} \right] \]  

(16)

Wind speed of 1 m s\(^{-1}\) was observed at the rear surface during laboratory measurement. It was due to the air stream from the air conditioning unit that controlled the laboratory air temperature. So, the heat transfer coefficient from this being calculated from an expression by McAdams [8]:

\[ h_s = 5.7 + 3.8V_w \]  

(17)

An expression for the radiation heat transfer coefficient between the insulated backing and the indoor environment was again evaluated from Duffie and Beckman [6]:

\[ h_{sa} = \sigma e_a [(T_a + 273)^2 + (T_a' + 273)^2](T_a + T_a' + 546) \]  

(18)

Heat transfer within the triple-layer cover was considered as follows. For heat transfer from within the sealed cover system, both natural convection and radiation heat transfer between the layers were assumed. Due to the thinness of the polycarbonate partitioning within the cover system, the heat transfer by conduction through the partitions (both vertical and horizontal) would be small compared to that by convection and radiation, and therefore was assumed to be negligible.

While radiation heat transfer was treated in a similar way to that already described, the Nusselt number for natural convection between the partitions within the cover system was evaluated using a correlation by El-Sherbiny et al. [10]:

\[ Nu = \left\{ \frac{0.0605 \text{Ra}^{1/3}}{1 + \left[ \frac{0.104 \text{Ra}^{0.294}}{1 + (6310/\text{Ra})^{1/2}} \right]} \right\}^{1/3} \]  

(19)

where

\[ Ra = \frac{g\beta A T d/Pr}{v^2} \]  

(20)

3. Laboratory testing

3.1. The test system

A vertical section of the collector arrangement was tested using a laboratory solar simulator, with the test system being configured as shown in Fig. 3. The air heating solar collector as tested was comprised of the following components:

(i) A 2 m long section of British Steel 'long rib 1000' profiled cladding with a profile depth of 0.03 m; the steel surface was finished with a black Prf2 coating, capable of withstanding a surface temperature of 120°C.

(ii) A backing of 0.1 m thick fibre-glass insulation.

(iii) A transparent cover consisting of triple-layer polycarbonate rigid sheeting ('Corotherm'), each layer separated by a spacing of 0.007 m.

A spacing of 0.1 m was allowed between the top of the steel profile and the base of the polycarbonate cover, the sides being enclosed by insulated plates. Electrically-heated air was supplied to the collector at the base via ductwork and a specially designed plenum. A similar arrangement was constructed at the top of the collector for removal of air. The temperature of the fan-induced air supply could be varied so as to generate standard efficiency characteristics.
Solar irradiance was provided by four 1000 W Compact Source Iodide (CSI) lamps, arranged in a square array. The position of the lamps within the array could be adjusted so as to achieve a maximum uniformity of irradiance on the test surface. Only the central 1 m² section of collector was used for testing so as to permit some flow development in the duct as would be expected in a full scale system. The average irradiance on the test surface was measured to be 407±58 W m⁻². This corresponds to an uncertainty of ±14.3%. The irradiance variation on the test surface was found to be 16% (coefficient of variation). Though this exceeds the value of 3% stipulated in the ASHRAE Standard for commercial collector testing [12], it is considered to be adequate in this study for the purposes of estimating collector characteristics, and for model validation.

The effect of a wind across the external surface of the collector cover was simulated using a set of three fans mounted at the collector base. These generated an average surface wind speed of 3.5 m s⁻¹. The difference in air temperature across the collector inlet and outlet was measured using K-type thermocouples arranged in a thermopile, comprised of six radiation-shielded junctions at each end of the duct. A similar thermopile was used to measure the air temperature difference between the collector inlet and the environment surrounding the collector. This arrangement gave an overall accuracy in temperature difference measurements of ±0.1°C. Surface temperatures throughout were measured using K-type thermocouples so as to give the average temperatures of plate and cover surfaces. Duct air mass flow rates were measured at collector inlet and outlet by performing traverses with a hot-wire anemometer giving an accuracy of ±0.01 kg s⁻¹. The entire test system was housed within an air-conditioned laboratory, which maintained a surrounding ambient temperature of 18°C ± 2°C throughout the tests.

3.2. Experimental results

At the start of each test, the fan at the outlet was switched on, followed by the solar simulator lamps; the system would then be allowed about 30 minutes to reach thermal equilibrium. The air velocity at each traverse position at both the inlet and the outlet would then be recorded manually using a hot-wire anemometer from which inlet and outlet air mass flow rates could be calculated. A further 30 minutes was then allowed before the first set of measurements was taken.

For each measurement set, the following data were recorded with a data-logger over a period of 4 minutes, from which the average could then be taken when determining the solar collector characteristic:

- inlet-outlet air temperature difference;
- inlet-ambient air temperature difference;
- the average temperatures of inner and outer cover surfaces.
- the absorber and the rear face of the insulation;
- inlet air temperature.

After each set of measurements, the inlet air temperature was increased and 30 minutes was allowed for steady-state conditions to be reached again prior to recording the next set of measurements. A total of eight sets of measurements were taken for each test. Results were plotted on a standard graph of efficiency (η) against (t₁ - t₂) / t₁, from which the overall collector heat transfer coefficient (Uc) and the heat removal factor (F̃) could be evaluated.

Three efficiency characteristics were obtained. They are presented in Fig. 4(a)–(c), for the conditions stated. From these characteristics, and using a value for (τ₀'), of 0.68 as
of additional edge effect heat losses that are present in the test system. Corrected $U_L$ and $F_R$ values for design use are presented later, but firstly the experimental results as found were used to validate the model of collector performance presented in Section 4. This is described next.

4. Model validation

In the laboratory test system, routes for additional heat loss from the collector test unit were identified as:

(i) conduction along the profiled absorber out of the considered control volume along the axis of fluid flow; and

(ii) conduction of heat through the edge plate extensions and its dissipation to the laboratory environment by convection and radiation.

By using second order differential equations and substituting the appropriate values for temperatures and wind speeds, the magnitude of the losses was estimated, and the findings used to correct predictions from the model (by Ho) presented in this paper, and also the model by Duffie and Beckman [6].

For the three collector operating conditions tested, Table I presents a comparison between the experimentally-measured values for $U_L$ and $F_R$ and those predicted by the Ho model and by the Duffie and Beckman model (for the case of ambient temperature at the front and rear of the collector both being equal to 18°C). It can be seen that the Ho model predicts a value for $U_L$ which is reasonably close to that measured, whereas the Duffie and Beckman model consistently overpredicts this value due to the assumption of an infinitely thin, infinitely conducting collector cover. Predictions for $F_R$ from the Ho model are also in better agreement with measured results than are those from the Duffie and Beckman model.

Further validation and intermodel comparison was carried out by comparing the measured temperature values for the collector cover (inner and outer) and absorber surfaces ($T_1$, $T_2$, $T_3$ in Fig. 2) with those predicted iteratively from the models. This is shown in Fig. 5, in which perfect agreement between measured and predicted surface temperatures would appear as a straight line through the origin with a gradient and a correlation coefficient both of unity. The lines shown in Fig. 5 correspond to linear regressions fitted to the measurements versus prediction results of the Ho and of the Duffie and Beckman models. Inspection shows that the Ho model

<table>
<thead>
<tr>
<th>Measurement conditions</th>
<th>Measured</th>
<th>Predicted by models of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ho</td>
<td>Duffie and Beckman [6]</td>
</tr>
<tr>
<td>$m=0.134 \text{ kg s}^{-1}, \eta=406.7 \text{ W m}^{-2}$</td>
<td>$12.2$</td>
<td>$8.5$</td>
</tr>
<tr>
<td>$m=0.203 \text{ kg s}^{-1}, \eta=406.7 \text{ W m}^{-2}$</td>
<td>$10.7$</td>
<td>$8.3$</td>
</tr>
<tr>
<td>$m=0.202 \text{ kg s}^{-1}, \eta=406.7 \text{ W m}^{-2}$</td>
<td>$12.6$</td>
<td>$8.4$</td>
</tr>
</tbody>
</table>

Table I

Comparison of measured $U_L$ and $F_R$ values with prediction, based on models of Ho and of Duffie and Beckman
Fig. 5. Measurement vs. prediction by Ho (●) and by Duffie and Beckman (▲) with linear regression fit. --- Regression fit (measured vs. Ho's prediction). --- Regression fit (measured vs. Duffie and Beckman's prediction).

This gives better agreement to measurement than does the Duffie and Beckman model, with correlation coefficients of 0.99 (Ho) and 0.86 (Duffie and Beckman).

5. Expected full-scale performance

The model presented in this paper was then used to estimate those values for FR, UL and energy collection rate that could be expected when the collector forms an integral part of a building envelope. To do this, it is necessary to adopt a suitable set of design conditions for both the climate and the building. It is assumed that the building is a leisure centre of lightweight construction, with an internal air temperature of 16°C (in the main sports hall). It is situated in the UK at latitude 51.7°N with a winter external design air temperature of -3°C. The collector is 5 m wide by 5 m high, and is part of the south-facing vertical wall; it receives a total daily mean solar irradiance of 130 W m⁻² (averaged from 08:00 to 16:00 h) [13].

For the outdoor environment, the convection coefficient at the outer surface of the transparent cover was evaluated using the following expression [6]:

\[ h_1 = 8.5 \frac{v^{0.6}}{L_w} \]  

(21)

where the terms are defined in the Nomenclature. A value for \( L_w \) was determined by assuming the leisure centre dimensions to be 34 m by 37 m by 7.1 m high (values typical for this type of building) [14]. If the leisure centre is located in an urban area then the following expression can be used to estimate a value for \( V_p \) at the mid-height of the building (3.55 m):

\[ V_p = V_m K_v \alpha \]  

(22)

where \( K_v \) and \( \alpha \) take values of 0.35 and 0.25, respectively. This gives a value for \( V_p \) of 2.2 m s⁻¹ and a corresponding value for \( h_1 \) of 4.1 W m⁻² °C⁻¹. For the evaluation of radiation loss from the outer cover surface, an equivalent sky temperature was used, calculated from the following correlation [15]:

\[ t_s = t_s^* - 6 \]  

(23)

The model presented in Section 2.1 (the Ho model) has been derived for the situation where air temperatures at the front and rear of the collector differ, that is, where \( t_s \neq t_s^* \). However, the Hottel-Whillier-Bliss analysis of solar collector performance is based on an energy balance in which \( t_s \) and \( t_s^* \) are always assumed equal and it is upon this equality that the standard collector parameters UL and FR are based. By setting \( t_s \) equal to \( t_s^* \) in the Ho model, the following expressions are derived for UL and FR for the covered profiled steel collector:

\[ U_L = \frac{K(U_s' + U_b')(h_s h_3 + h_s h_2 + K h_s h_3) + U_s' U_b'(h_s + K h_s)}{K(h_s h_23 + h_s U_s' + K h_s h_23 + h_s h_3)} \]  

(24)

\[ F_R = \frac{K(h_s h_23 + h_s U_s' + K h_s h_23 + h_s h_3)}{(U_s' + K h_s h_23 + h_s)(U_b + K h_s h_23) - K^2 h_s h_3} \]  

(25)

While the authors are currently pursuing work based upon simulation studies of building and collector performance in which front and rear ambient temperatures differ (the more realistic situation), in order to make a direct comparison with existing literature, typical values for \( U_L \) and \( F_R \) are determined as follows for the case of \( t_s = t_s^* \). An equivalent surrounding ambient air temperature \( t_s^* \) for the wall-integrated collector was estimated using:

\[ t_s^* = \frac{U_s' t_s + U_b t_b}{U_s' + U_b} \]  

(26)

in which \( t_s = -3^\circ \text{C}, \ t_s^* = 16^\circ \text{C}, \ U_s' = 2.6 \text{ W m}^{-2} \text{ °C}^{-1} \) and \( U_b = 1.2 \text{ W m}^{-2} \text{ °C}^{-1} \; \text{°C}^{-1} \); this gives a value for \( t_s^* \) in place of \( t_s \) and \( t_s^* \) in the Ho model leading to equivalent values for \( U_L \) and \( F_R \).
and $F_w$ of approximately 4.0 W m$^{-2}$ °C$^{-1}$ and 0.71, respectively. These values at present give only the approximate performance of wall-integrated covered profiled steel collectors. For a ($\tau_a$) value of 0.68 and an irradiance of 130 W m$^{-2}$, an overall efficiency of 0.43 is obtained, leading to a heat supply rate from a 5 m by 5 m collector area of 1.4 kW for the conservative conditions assumed.

6. Conclusions

A validated mathematical model for predicting the thermal performance of a profiled steel air heating solar collector covered with a multi-layer polycarbonate cover system has been presented. The model is able to predict the performance when front and rear ambient temperature differ, and thus can be used for estimating the performance of such a collector system when installed as an integral wall or roof of a building. For the condition of front and rear temperatures being equal, the model presented can be compared with one given by Duffie and Beckman [6], and it has been shown to give the better agreement with measurement.

A set of standard collector parameters characterizing the performance of the proposed solar collector system as if part of a full-scale building has also been presented for approximate performance estimation and comparison with the literature, for the condition where air temperatures at the front and rear of the collector are assumed to be equal.

The authors are conducting simulation studies for the more realistic situation in which front and rear ambient temperatures differ, in order to fully assess the potential of such building-integrated solar collectors in the UK climate.

7. Nomenclature

- $a$: a parameter, defined in Eq. (21) and Ref. [15] (-)
- $A$: cover area (m$^2$)
- $A_3$: absorber area (accounting for corrugations) (m$^2$)
- $A_4$: edge plate area (1 side) that form the flow channel (m$^2$)
- $AR$: aspect ratio of the compartment in the cover system (-)
- $C$: specific heat capacity of channel fluid (J kg$^{-1}$ °C$^{-1}$)
- $d$: spacing between layers in the cover system (m)
- $g$: acceleration due to gravity = 9.81 m s$^{-2}$
- $h_1$: convection heat transfer coefficient between the outer cover surface and the front ambient environment (W m$^{-2}$ °C$^{-1}$)
- $h_2$: convection heat transfer coefficient between channel fluid and the inner cover surface (W m$^{-2}$ °C$^{-1}$)
- $h_3$: convection heat transfer coefficient between channel fluid and the absorber surface (W m$^{-2}$ °C$^{-1}$)
- $h_4$: convection heat transfer coefficient between the rear surface and the rear ambient environment (W m$^{-2}$ °C$^{-1}$)
- $h_r$: radiation heat transfer coefficient between the outer cover surface and the front ambient environment (W m$^{-2}$ °C$^{-1}$)
- $h_{r23}$: radiation heat transfer coefficient between absorber and cover inner surfaces (W m$^{-2}$ °C$^{-1}$)
- $h_{r4}$: radiation heat transfer coefficient between rear surface and rear environment (W m$^{-2}$ °C$^{-1}$)
- $h_{r11a}$: natural convection heat transfer coefficient between outer and middle cover layers (W m$^{-2}$ °C$^{-1}$)
- $h_{r11b}$: radiation heat transfer coefficient between outer and middle cover layers (W m$^{-2}$ °C$^{-1}$)
- $h_{r12a}$: natural convection heat transfer coefficient between middle and inner cover layers (W m$^{-2}$ °C$^{-1}$)
- $h_{r12b}$: radiation heat transfer coefficient between middle and inner cover layers (W m$^{-2}$ °C$^{-1}$)
- $I$: irradiance on the cover (W m$^{-2}$)
- $k_a$: conductivity of air
- $K$: area enhancement factor for corrugation ($A_3/A$) (-)
- $K_{conv}$: equivalent heat transfer coefficient of the cover system (W m$^{-2}$ °C$^{-1}$)
- $K_a$: a parameter, defined in Eq. (21) and Ref. [15] (-)
- $L$: length of collector (m)
- $L'$: characteristic plate dimension, average of length and width (m)
- $L_c$: cube root of building volume (m$^3$)
- $m$: mass flow rate of channel fluid (kg s$^{-1}$)
- $Nu$: Nusselt number (-)
- $Nu_{tid}$: fully-developed Nusselt number (-)
- $Pr$: Prandtl number
- $Ra$: Rayleigh number (-)
- $Re$: Reynolds number (-)
- $T_1$: cover front surface temperature (°C)
- $T_2$: inner surface temperature of cover (°C)
- $T_3$: absorber surface temperature (°C)
- $T_4$: rear surface temperature (°C)
- $T_{ax}$: front ambient air temperature (°C)
- $T_{ar}$: rear ambient air temperature (°C)
- $T_{aw}$: weighted equivalent ambient temperature surrounding collector (°C)
- $T_f$: fluid temperature in flow channel (°C)
- $T_{in}$: air inlet temperature (°C)
- $T_{out}$: air outlet temperature (°C)
- $T_s$: sky temperature (°C)
- $U_b$: heat loss coefficient from the rear of the collector, taken from the absorber surface to the rear ambient environment (W m$^{-2}$ °C$^{-1}$)
Appendices


$U_e$ edge insulation heat loss coefficient
($W\ m^{-2}\ °C^{-1}$)

$U_i$ heat loss from the front of the collector through the cover alone, taken from the inner cover surface to front ambient environment ($W\ m^{-2}\ °C^{-1}$)

$U_i'$ heat loss coefficient from the front of the collector, inclusive of heat loss from edge plates, taken from inner cover surface to front ambient environment ($U_i + 2(U_{Ac}/LW)$ ($W\ m^{-2}\ °C^{-1}$))

$V_m$ mean wind speed at height of 10 metre ($m\ s^{-1}$)

$V_p$ cover surface wind speed ($m\ s^{-1}$)

$V_r$ rear surface wind speed ($m\ s^{-1}$)

$W$ width of collector module ($m$)

$z$ height above ground ($m$)

$\Delta T\;_f$ change of fluid temperature per metre of channel

$\frac{dy}{dz}$ length of flow channel in the $y$-direction, i.e.

Greek letters

$(ta)_e$ effective transmittance–absorptance product ($-$)

$\beta$ volumetric thermal expansion coefficient ($°C^{-1}$)

$\sigma$ Stefan–Boltzmann Constant = $5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$

$\varepsilon_i$ emissivity of cover layers ($-$)

$\varepsilon_s$ emissivity of absorber surface (fluid channel side) ($-$)

$\varepsilon_r$ rear surface emissivity ($-$)

$\nu$ kinematic viscosity ($m^2\ s^{-1}$)

Acknowledgements

The authors are grateful to Professor M.G. Hutchins of Oxford Brookes University for the use of the Bruker IFS 66/S spectrometer for spectral transmission and absorption measurements.

References

A2. Thermopile calibration

Thermopiles were constructed to measure collector inlet-outlet and inlet-ambient temperature differences. Each thermocpile was calibrated as follows:

- The 'hot' thermo-junctions were placed in a waterproofed plastic bag, the 'cold' junctions in another.

- The 'cold' junctions (waterproofed in a plastic bag) were put into a water bath submerged with ice while the 'hot' end (also waterproofed with a plastic bag) was filled with warm water at an initial temperature of 30°C \( (t_h - t_c) \) or 40°C \( (t_i - t_c) \). A mercury thermometer was placed in each water bath to obtain the water temperature. A magnetic stirrer was used in each water bath to create an even temperature through a water bath. While the thermometer in the 'hot' water bath was to measure its temperature for the calibration, the thermometer in the 'cold' water bath was to ensure its temperature being maintained at 0°C.

- A reading in voltage (in milliVolt) was then obtained for the corresponding 'hot' water bath temperature.

- Temperature of the 'hot' water bath was reduced by putting ice into the water bath. This enable corresponding voltages for lower water bath temperatures to be obtained.

- For each thermopile system, a graph of temperature difference between the 'hot' and 'cold' water baths against the corresponding voltage (in milliVolt) measured across the thermopile system was then plotted. The conversion coefficient for each thermopile was obtained from the gradient of the graph (Figure A2.1, A2.2 and Table A2.1).
Figure A2.1: Characteristic of thermopile for measuring collector inlet-ambient temperature difference

Figure A2.2: Characteristic of thermopile for measuring collector inlet-outlet temperature difference

<table>
<thead>
<tr>
<th>Graph</th>
<th>Conversion coefficient (mV°C⁻¹)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_i - t_o )</td>
<td>0.236837</td>
<td>0.9996</td>
</tr>
<tr>
<td>( t_o - t_i )</td>
<td>0.240333</td>
<td>0.9997</td>
</tr>
</tbody>
</table>

Table A2.1: Thermopiles' conversion coefficient and the coefficient of variation obtained
A3. Estimation of the optimum lamp-to-lamp spacings for the solar simulator

Estimation of the irradiance distribution was carried out before the laboratory thermal performance measurement of the solar collector was conducted. This was to ensure that the the best distribution of simulated irradiance source was obtained.

Data published (Krusi and Schmid, 1983) for irradiance distribution of a single CSI 1000W lamp (held at 2.66 m perpendicular above horizontal surface) were used for the estimation. It was presented as in Figure A3.1.

![Figure A3.1: The illumination pattern of a single CSI 1000W lamp (Krusi and Schmid, 1983)](image)

It is known (Parker, 1991) that the irradiance output (in Wm\(^{-2}\)) is proportional to the reciprocal of the square of the distance from the source (also known as inverse-square law), the following was true:

\[
I_s = \frac{K_s}{d_s^2} \quad \text{...(A3.1)}
\]

where

\[
\begin{align*}
  d_s & = \text{distance between a CSI lamp to illuminating point at angle } \theta \quad \text{(m)} \\
  I_s & = \text{Irradiance on the surface at angle } \theta \quad \text{(Wm}^{-2}\text{)}
\end{align*}
\]
Appendices

\[ K_s = \text{a constant, dependent on angle } \theta \text{ (W)} \]

\[ \theta = \text{angle between the normal to the centre of the CSI lamp and the line from the centre of the lamp to the point considered (°)} \]

---

Figure A3.2: Format converted from data using equation A3.1 and the fitted curve

Data on irradiance from Krusi and Schmid (1983) were converted to angle dependent constants, \( K_s \), using Equation A3.1 and their corresponding angle, \( \theta \), so that a correlation could be formulated by curve fitting (Figure A3.2). The reason is that the constant \( K_s \) remains constant for a particular angle \( \theta \). Fifteen pairs of constants \( K_s \) and angles \( \theta \) were taken (as seen in Figure A3.1) for formulating the correlation. They were then substitute into a polynomial function of the following format to form 15 simultaneous equations:

\[ K_s = a_0 + a_1 \theta + a_2 \theta^2 + a_3 \theta^3 + \ldots + a_{14} \theta^{14} \] ... (A3.2)

A matrix in the following format was then formed by the 15 simultaneous equations:

\[
\begin{bmatrix}
I_{\theta 1} \\
I_{\theta 2} \\
\vdots \\
I_{\theta 15}
\end{bmatrix} = 
\begin{bmatrix}
1 & \theta_1 & \ldots & \theta_1^{14} \\
1 & \theta_2 & \ldots & \theta_2^{14} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \theta_{15} & \ldots & \theta_{15}^{14}
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_{14}
\end{bmatrix} \] ... (A3.3)

By multiplying the inverse matrix of the matrix on the right hand side of Equation A3.3 to both sides of the matrix equality, the following was formulated (Figure A3.2):
The above equation was then used in a spreadsheet to calculate the irradiance together with equation A3.1. The equation enables the calculation of irradiance on a surface perpendicular held from a single CSI lamp.

The solar simulator (Figure A3.3) consisted of 4 CSI 1000W lamps with each lamp situated in a corner of a square lamp array which centre aligned with the centre of the cover (so that a symmetrical irradiance distribution would be expected on the four quadrant of the cover surface). The solar simulator was set up that the simulated solar irradiance might be intercepted perpendicular onto the collector cover system.

The resultant irradiance received at a point of the cover surface from the solar simulator is the algebraic sum of the irradiance from each lamp received at the point. The irradiance distribution on the collector cover surface was then estimated in a matrix of 1600 cells (40 row by 40 column) on a spreadsheet. The mean irradiance on the surface and the variation in terms of coefficient of variation (CV) were the calculated. The optimum lamp-lamp spacing (which
produce the most even irradiance distribution) is obtained when the resultant CV is minimum. Table A3.1 presents the results.

![Figure A3.3: Estimated irradiance distribution on the 1m by 1m collector cover produced by the solar simulator with a simulator-cover distance of 4m and a lamp-lamp spacing of 0.72m](image)

<table>
<thead>
<tr>
<th>Lamp-cover distance, ( H ) (m)</th>
<th>Lamp-lamp spacing, ( d ) (m)</th>
<th>Mean irradiance, ( \text{CV} ) (Wm(^{-2}))</th>
<th>CV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.60</td>
<td>625.0</td>
<td>14.24</td>
</tr>
<tr>
<td>4</td>
<td>0.72</td>
<td>433.8</td>
<td>5.77</td>
</tr>
<tr>
<td>5</td>
<td>0.84</td>
<td>317.0</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Table A3.1: Results obtained from spreadsheet

References


A4. Estimation of additional heat loss from the collector test module

The extra heat loss mechanism consists of the following (Figure A6.1):

- heat loss along absorber surface to edge of collector (normal to air flow direction);
- axial conduction along absorber surface to associated ductwork (same direction to air flow); and,
- heat loss from the edge of collector to simulated wind speed (acting as a fin).

![Diagram showing extra heat flow direction from the collector](image)

Figure A6.1: Extra heat flow direction from the collector

The extra heat loss mechanism due to axial conduction out of the 'control volume' (collector system considered) was estimated by second order differential equations.
Extra conduction heat loss from absorber to associated ductwork

Figure A6.2: Consideration of parameters for extra heat flow in x-direction

\[
- \left\{ kA_x \frac{d^2 T_x}{dx^2} \Delta x \right\} + \left\{ h_3 W \Delta x [ T_x - T_\infty ] + \frac{k_1}{d_2} W \Delta x [ T_\infty - T_x ] \right\} = 0
\]

\[
- \frac{kA_x}{W} \frac{d^2 T_x}{dx^2} + h_3 [ T_x - T_\infty ] + \frac{k_1}{d_2} [ T_\infty - T_4 ] = 0
\]

\[
\frac{kA_x}{W} \frac{d^2 T_x}{dx^2} - \left( h_3 + \frac{k_2}{d_2} \right) T_x = - \left( h_3 T_\infty + \frac{k_2}{d_2} T_4 \right)
\]

Find general solution \( T_x \) of

\[
\frac{d^2 T_x}{dx^2} - \frac{W}{kA_x} \left( h_3 + \frac{k_2}{d_2} \right) T_x = 0
\]

Compare coefficient \( T_x'' + aT_x' + bT_x = 0 \), we get

\[ a = 0 \text{ and } b = - \frac{W}{kA_x} \left( h_3 + \frac{k_2}{d_2} \right) \]

Since,
\[ r = \frac{-a \pm \sqrt{a^2 - 4b}}{2} \]
\[ = \pm \frac{1}{2} \sqrt{\frac{4W}{kA_x} \left( \frac{h_3 + k_2}{d_2} \right)} \]
\[ = \pm \sqrt{\frac{W}{kA_x} \left( \frac{h_3 + k_2}{d_2} \right)} \]

So the roots are:
\[ r_1 = \sqrt{\frac{W}{kA_x} \left( \frac{h_3 + k_2}{d_2} \right)} \quad \text{and} \quad r_2 = -\sqrt{\frac{W}{kA_x} \left( \frac{h_3 + k_2}{d_2} \right)} \]

Therefore, the general solution is:
\[ T_h = C_1 \exp \left[ x \sqrt{\frac{W}{kA_x} \left( \frac{h_3 + k_2}{d_2} \right)} \right] + C_2 \exp \left[ -x \sqrt{\frac{W}{kA_x} \left( \frac{h_3 + k_2}{d_2} \right)} \right] \quad \ldots (A6.2) \]

To find particular solution of \( \frac{kA_x}{W} \frac{d^2 T_p}{dx^2} - \left( h_3 + \frac{k_2}{d_2} \right) T_p = -\left( h_3 T_m + \frac{k_2}{d_2} T_4 \right) \), the following was tried:
\[ T_p = C \]
\[ \frac{dT_p}{dx} = 0 \]

and
\[ \frac{d^2 T_p}{dx^2} = 0 \]

So,
\[ \left( h_3 + \frac{k_2}{d_2} \right) C = \left( h_3 T_m + \frac{k_2}{d_2} T_4 \right) \]
\[ \left( h_3 T_m + \frac{k_2}{d_2} T_4 \right) \]
\[ C = \frac{\left( h_3 + \frac{k_2}{d_2} \right)}{\left( h_3 + \frac{k_2}{d_2} \right)} \]
and hence

\[ T_p = \frac{(h_3 T_\infty + k_2 / d_2) T_x}{h_3 + k_2 / d_2} \]

Therefore, the general solution of

\[ \frac{kA_x}{W} \frac{d^2 T_x}{dx^2} - \left( h_3 + \frac{k_2}{d_2} \right) T_x = -\left( h_3 T_\infty + \frac{k_2}{d_2} T_x \right) \]

is

\[ T = T_h + T_p \]

\[ T_x = C_1 \exp(xr_1) + C_2 \exp(-xr_1) + \frac{(h_3 T_\infty + k_2 / d_2) T_x}{h_3 + k_2 / d_2} \]

Since \( \sinh(rx) \) and \( \cosh(rx) \) can be constructed by the combination of \( \exp(-rx) \) and \( \exp(rx) \). The following alternative forms is expressed:

\[ T_x = C_1 \cosh(xr_1) + C_2 \sinh(xr_1) + \frac{(h_3 T_\infty + k_2 / d_2) T_x}{h_3 + k_2 / d_2} \]

Boundary condition 1: \( x = L_1 \), \( T_x = T_{nip} \)

\[ T_{nip} = C_1 \cosh(r_1 L_1) + C_2 \sinh(r_1 L_1) + \frac{(h_3 T_\infty + k_2 / d_2) T_x}{h_3 + k_2 / d_2} \] \quad \text{...(A6.3)}

Boundary condition 2: \( x = 0 \), \( T_x = T_h \)

\[ T_h = C_1 + \frac{(h_3 T_\infty + k_2 / d_2) T_x}{h_3 + k_2 / d_2} \]

\[ C_1 = T_h - \frac{(h_3 T_\infty + k_2 / d_2) T_x}{h_3 + k_2 / d_2} \] \quad \text{...(A6.4)}
Let  

\[ C_3 = \frac{(h_3 T_\alpha + k_2/\bar{d}) T_4}{(h_3 + k_2/\bar{d})} \]  

..(A6.5)

Equation (A6.3) becomes,

\[ T_{up} = C_1 \cosh(r_1 L_1) + C_2 \sinh(r_1 L_1) + C_3 \]  

...(A6.6)

and equation (A6.4) becomes,

\[ C_1 = T_b - C_3 \]  

...(A6.7)

Substitute equation (A6.7) into (A6.6),

\[ T_{up} = (T_b - C_3) \cosh(r_1 L_1) + C_1 \sinh(r_1 L_1) + C_3 \]

\[ C_2 = \frac{(T_{up} - C_3) - (T_b - C_3) \cosh(r_1 L_1)}{\sinh(r_1 L_1)} \]  

...(A6.8)

Therefore,

\[ T_x = (T_b - C_3) \cosh(x r_1) + \frac{(T_{up} - C_3) - (T_b - C_3) \cosh(r_1 L_1)}{\sinh(r_1 L_1)} \sinh(x r_1) \]

\[ + \frac{(h_3 T_\alpha + k_2/\bar{d}) T_4}{(h_3 + k_2/\bar{d})} \]  

...(A6.9)

Differentiate the above to yield:

\[ T_x' = r_1 (T_b - C_3) \sinh(x r_1) + r_1 \frac{(T_{up} - C_3) - (T_b - C_3) \cosh(r_1 L_1)}{\sinh(r_1 L_1)} \cosh(x r_1) \]  

...(A6.10)

\[ Q_{ext} = -k A_x \left. \frac{dT}{dX} \right|_{x=0} \]

\[ Q_{ext} = -k A_x r_1 \frac{(T_{up} - C_3) - (T_b - C_3) \cosh(r_1 L_1)}{\sinh(r_1 L_1)} \]
Axial heat loss between absorber and edge plate (direction normal to that of air flow)

![Diagram of heat flow between absorber and edge plate](image)

Figure A6.3: Consideration of parameters for extra heat flow in y-direction

**et rate of heat loss**
- by conduction in y-direction from volume element \( \Delta y \)
- by convection & conduction from volume element \( \Delta y \)

**et rate of heat gain**
- by solar energy to volume element \( \Delta y \)

\[
-kA_x \frac{d^2 T_y}{dy^2} + \left( h_y + k_y \right) \frac{T_y}{d_y} + \left( h_y + k_y \right) T_y = \left( h_y + k_y \right) T_y = -\left( h_y + k_y \right) T_y + \left( h_y + k_y \right) T_y + I(\tau \alpha)_e
\]

The general solution for \( \frac{kA_x}{L} \frac{d^2 T_y}{dy^2} = (h_y + k_y)T_y = 0 \) would be:

\[
T_y = C_1 \cosh \left[ y \sqrt{\frac{L}{kA_x}} \left( h_y + \frac{k_y}{d_y} \right) \right] + C_2 \sinh \left[ y \sqrt{\frac{L}{kA_x}} \left( h_y + \frac{k_y}{d_y} \right) \right]
\]

To find particular solution, of \( \frac{kA_x}{L} \frac{d^2 T_y}{dy^2} = (h_y + k_y)T_y = -L \left( h_y + \frac{k_y}{d_y} \right) T_y + I(\tau \alpha)_e \),

try

\[
T_p = C
\]
\[
\frac{dT_p}{dy} = 0
\]

and

\[
\frac{d^2 T_p}{dy^2} = 0
\]

\[-(h_3 + \frac{k_2}{d_2}) C = I(\tau \alpha) e - (h_3 T_\infty + \frac{k_2}{d_2} T_4)\]

\[C = \frac{(h_3 T_\infty + \frac{k_2}{d_2} T_4)}{(h_3 + \frac{k_2}{d_2})} - \frac{I(\tau \alpha) e}{(h_3 + \frac{k_2}{d_2})}\]

\[T_p = C_3 - \frac{I(\tau \alpha) e}{(h_3 + \frac{k_2}{d_2})}\]

Therefore,

\[T_y = C_4 \cosh \left[ y \sqrt{\frac{L}{k d_2}} \left( h_3 + \frac{k_2}{d_2} \right) \right] + C_5 \sinh \left[ y \sqrt{\frac{L}{k d_2}} \left( h_3 + \frac{k_2}{d_2} \right) \right] - \frac{I(\tau \alpha) e}{(h_3 + \frac{k_2}{d_2})} + C_3\]

Boundary condition 1: \(y = \frac{W}{2}, T_y = T_{up},\)

\[T_{up} = C_4 \cosh \left[ \frac{W}{2} \sqrt{\frac{L}{k d_2}} \left( h_3 + \frac{k_2}{d_2} \right) \right] + C_5 \sinh \left[ \frac{W}{2} \sqrt{\frac{L}{k d_2}} \left( h_3 + \frac{k_2}{d_2} \right) \right] - \frac{I(\tau \alpha) e}{(h_3 + \frac{k_2}{d_2})} + C_3\]

...(A6.12)

Boundary condition 2: \(y = 0, T_y = T_b,\)

\[T_b = C_4 - \frac{I(\tau \alpha) e}{h_3 + \frac{k_2}{d_2}} + C_3\]

\[C_4 = T_b + \frac{I(\tau \alpha) e}{h_3 + \frac{k_2}{d_2}} - C_3\]

...(A6.13)
Substitute (A6.13) into (A6.12),

\[ T_{np} = \left[ T_{n} + \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} - C_{3} \right] \cosh \left[ \frac{W}{2} \sqrt{\frac{L}{kA_{x}}} \left( \frac{h_{3} + k_{2}}{d_{2}} \right) \right] + C_{3} \sinh \left[ \frac{W}{2} \sqrt{\frac{L}{kA_{x}}} \left( \frac{h_{3} + k_{2}}{d_{2}} \right) \right] - \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} + C_{3} \]

...(A6.14)

Let \( r_{3} = \sqrt{\frac{L}{kA_{x}}} \left( \frac{h_{3} + k_{2}}{d_{2}} \right) \)

Equation (A6.14) becomes,

\[ T_{np} = \left[ T_{n} + \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} - C_{3} \right] \cosh \left( \frac{r_{3}W}{2} \right) + C_{3} \sinh \left( \frac{r_{3}W}{2} \right) \]

\[ - \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} + C_{3} \]

\[ C_{3} \sinh \left( \frac{r_{3}W}{2} \right) = \left( T_{np} - C_{3} \right) - \left[ T_{n} + \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} - C_{3} \right] \cosh \left( \frac{r_{3}W}{2} \right) + \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} \]

\[ C_{3} = \frac{\left( T_{np} + \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} - C_{3} \right) \cosh \left( \frac{r_{3}W}{2} \right)}{\sinh \left( \frac{r_{3}W}{2} \right)} \]

...(A6.15)

Let

\[ C_{6} = \frac{I(\tau \alpha)}{h_{3} + k_{2}/d_{2}} - C_{3} \]

Equation (A6.15) becomes

\[ C_{5} = \frac{(T_{np} + C_{6}) - (T_{n} + C_{6}) \cosh \left( \frac{r_{3}W}{2} \right)}{\sinh \left( \frac{r_{3}W}{2} \right)} \]
and equation (A6.13) becomes

\[ C_4 = T_h + C_6 \]

Therefore,

\[ T_v = \left( T_h + C_6 \right) \cosh(r_3 y) - C_6 \]

\[ - \left( T_h + C_6 \right) \cosh(\frac{r_3 W}{2}) \quad \sinh(r_3 y) \]

\[ \quad \sinh(\frac{r_3 W}{2}) \]

\[ \ldots (A6.16) \]

\[ T_v' = r_3 \left( T_h + C_6 \right) \sinh(r_3 y) + r_3 \left( T_{ip} + C_6 \right) \cosh(\frac{r_3 W}{2}) \quad \cosh(r_3 y) \]

\[ - \left( T_h + C_6 \right) \cosh(\frac{r_3 W}{2}) \quad \cosh(r_3 y) \]

\[ \ldots (A6.17) \]

\[ Q_{\text{profile-side}} = -kA_x \frac{dT}{dy}_{y=\frac{W}{2}} \]

\[ Q_{\text{profile-side}} = -kA_x \int_{r_3 W/2}^{r_3 W} \left( T_h + C_6 \right) \sinh(r_3 y) \]

\[ + \left( T_{ip} + C_6 \right) \cosh(\frac{r_3 W}{2}) \quad \cosh(r_3 y) \]

\[ \ldots (A6.18) \]

From edge plate-absorber contact to outside

\[ \text{Figure A6.4: Consideration of parameters for extra heat flow in } z \text{-direction} \]
Appendices

Net rate of heat loss by conduction in z-direction from volume element \( \Delta z \) + Net rate of heat loss by conduction & from collector air stream to element \( \Delta x \) = Net rate of heat gain by convection from duct air to element \( \Delta z \)

\[
-kA_x \frac{d^2 T_z}{dz^2} + U_x L[T_z - T_o] = h_x ' L[T_o - T_z]
\]

\[
-kA_x \frac{d^2 T_z}{dz^2} + U_x L[T_z - T_o] = h_x ' [T_o - T_z]
\]

\[
\frac{d^2 T_z}{dz^2} - \frac{L}{kA_x} (U_x + h_x ' T_z) = - \frac{L}{kA_x} (U_x t_o + h_x ' T_x)
\]

General solution should be:

\[
T_h = C_1 Cosh \left( z \sqrt{\frac{L}{kA_x}} (U_x + h_x ') \right) + C_8 Sinh \left( z \sqrt{\frac{L}{kA_x}} (U_x + h_x ') \right)
\]

For particular solution, let \( T_p = C, T_p ' = 0, T_p '' = 0 \)

\[
- \frac{L}{kA_x} (U_x + h_x ') C = - \frac{L}{kA_x} (U_x t_o + h_x ' T_o)
\]

\[
C = \frac{U_x t_o + h_x ' T_o}{U_x + h_x '}
\]

Therefore,

\[
T_p = \frac{U_x t_o + h_x ' T_o}{U_x + h_x '}
\]

Let \( r = \sqrt{\frac{L}{kA_x}} (U_x + h_x ')
\]

\[
T_z = C \cdot Cosh(r_z z) + C_8 Sinh(r_z z) + \frac{U_x t_o + h_x ' T_o}{U_x + h_x '}
\]

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Let \( C_g = \frac{U_c T_a + h_2' T_w}{U_c + h_2'} \)

Therefore,

\[
T_z = C_7 \cosh(r_4z) + C_8 \sinh(r_4z) + C_9 \quad \text{ ...(A6.19)}
\]

Boundary condition 1: \( z = \frac{L_2}{2}, T_z = T_e \)

\[
T_e = C_7 \cosh\left(\frac{r_4 L_2}{2}\right) + C_8 \sinh\left(\frac{r_4 L_2}{2}\right) + C_9 \quad \text{ ...(A6.20)}
\]

Boundary condition 2: \( z = L_2, T_z = T_{e1} \)

\[
T_{e1} = C_7 \cosh(r_4 L_2) + C_8 \sinh(r_4 L_2) + C_9 \quad \text{ ...(A6.21)}
\]

Equation (A6.20) \( \times \cosh(r_4 L_2) \),

\[
T_e \cosh(r_4 L_2) = C_7 \cosh\left(\frac{r_4 L_2}{2}\right) \cosh(r_4 L_2) + C_8 \sinh\left(\frac{r_4 L_2}{2}\right) \cosh(r_4 L_2)
+ C_9 \cosh(r_4 L_2) \quad \text{ ...(A6.22)}
\]

Equation (A6.21) \( \times \cosh\left(\frac{r_4 L_2}{2}\right) \),

\[
T_{e1} \cosh\left(\frac{r_4 L_2}{2}\right) = C_7 \cosh(r_4 L_2) \cosh\left(\frac{r_4 L_2}{2}\right) + C_8 \sinh(r_4 L_2) \cosh\left(\frac{r_4 L_2}{2}\right)
+ C_9 \cosh\left(\frac{r_4 L_2}{2}\right) \quad \text{ ...(A6.23)}
\]

Equation (A6.22) - Equation (A6.23),

\[
T_e \cosh(r_4 L_2) - T_{e1} \cosh\left(\frac{r_4 L_2}{2}\right) = C_8 \left[ \sinh\left(\frac{r_4 L_2}{2}\right) \cosh(r_4 L_2) - \sinh(r_4 L_2) \cosh\left(\frac{r_4 L_2}{2}\right) \right]
+ C_9 \left[ \cosh(r_4 L_2) - \cosh\left(\frac{r_4 L_2}{2}\right) \right]
\]

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\[ C_8 = \frac{T_c \cosh(r_4 L_z) - T_{10} \cosh\left(\frac{r_4 L_z}{2}\right) - C_9 \left[ \cosh(r_4 L_z) - \cosh\left(\frac{r_4 L_z}{2}\right) \right]}{\sinh\left(\frac{r_4 L_z}{2}\right) \cosh(r_4 L_z) - \sinh(r_4 L_z) \cosh\left(\frac{r_4 L_z}{2}\right)} \]

Equation (A6.20) \times \sinh(r_4 L_z),

\[ T_c \sinh(r_4 L_z) = C_7 \cosh\left(\frac{r_4 L_z}{2}\right) \sinh(r_4 L_z) + C_8 \sinh\left(\frac{r_4 L_z}{2}\right) \sinh(r_4 L_z) \]

\[ + C_9 \sinh(r_4 L_z) \]

\[ \ldots (A6.24) \]

Equation (A6.21) \times \sinh\left(\frac{r_4 L_z}{2}\right),

\[ T_c \sinh\left(\frac{r_4 L_z}{2}\right) = C_7 \cosh(r_4 L_z) \sinh\left(\frac{r_4 L_z}{2}\right) + C_8 \sinh(r_4 L_z) \sinh\left(\frac{r_4 L_z}{2}\right) \]

\[ + C_9 \sinh(r_4 L_z) \]

\[ \ldots (A6.25) \]

Equation (A6.24) - (A6.25),

\[ T_c \sinh(r_4 L_z) - T_{c0} \sinh\left(\frac{r_4 L_z}{2}\right) \]

\[ = C_7 \left[ \cosh\left(\frac{r_4 L_z}{2}\right) \sinh(r_4 L_z) - \cosh(r_4 L_z) \sinh\left(\frac{r_4 L_z}{2}\right) \right] \]

\[ + C_9 \left[ \sinh(r_4 L_z) - \sinh\left(\frac{r_4 L_z}{2}\right) \right] \]

\[ C_3 = \frac{T_c \sinh(r_4 L_z) - T_{c0} \sinh\left(\frac{r_4 L_z}{2}\right) - C_9 \left[ \sinh(r_4 L_z) - \sinh\left(\frac{r_4 L_z}{2}\right) \right]}{\cosh\left(\frac{r_4 L_z}{2}\right) \cosh(r_4 L_z) - \cosh(r_4 L_z) \cosh\left(\frac{r_4 L_z}{2}\right)} \]

For \( L_2 \leq z \leq L_1 + L_3, \)

\[ \text{rate of heat loss by conduction in } z \text{- direction from volume element } \Delta z \]

\[ + \text{rate of heat loss by convection, conduction and radiation from volume element } \Delta z = 0 \]

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where

\[ f = \text{metal frame's U-value (Wm}^{-2}\cdot{\circ}\text{C)} \]
\[ h_f = \text{combined convection and radiation heat transfer coefficient (Wm}^{-2}\cdot{\circ}\text{C)} \]

\[- \left\{ kA_z \frac{d^2 T_z}{dz^2} \Delta z \right\} + \left\{ U_f L \Delta z [T_z - T_a] + h_f L \Delta z [T_z - T_a] \right\} = 0 \]

\[
\frac{d^2 T_z}{dz^2} - \frac{L(U_f + h_f')}{kA_z} [T_z - T_a] = 0
\]

Let \( r_s = \sqrt{\frac{L(U_f + h_f')}{kA_z}} \)

\[ T_z = C_{10} \cosh(r_s z) + C_{11} \sinh(r_s z) + t_a \]

Boundary condition 1: \( z = L_2, T_z = T_{e1} \)

\[ T_{e1} = C_{10} \cosh(r_s L_2) + C_{11} \sinh(r_s L_2) + t_a \] \hspace{1cm} \text{(A6.26)}

Boundary condition 2: \( z = L_2 + L_3, T = T_{e2} \)

\[ T_{e2} = C_{10} \cosh[r_s (L_2 + L_3)] + C_{11} \sinh[r_s (L_2 + L_3)] + t_a \] \hspace{1cm} \text{(A6.27)}

Equation (A6.26) \( \times \cosh[r_s (L_2 + L_3)] - \text{Equation (A6.27)} \times \cosh(r_s L_2), \)

\[ C_{10} = \frac{T_{e1} \cosh[r_s (L_2 + L_3)] - T_{e2} \cosh(r_s L_2) + t_a [\cosh(r_s L_2) - \cosh[r_s (L_2 + L_3)]]}{\sinh[r_s (L_2 + L_3)] \cosh[r_s (L_2 + L_3)] - \cosh(r_s L_2) \sinh[r_s (L_2 + L_3)]} \] \hspace{1cm} \text{(A6.28)}

Equation (A6.26) \( \times \sinh[r_s (L_2 + L_3)] - \text{Equation (A6.27)} \times \sinh(r_s L_2), \)

\[ C_{11} = \frac{T_{e1} \sinh[r_s (L_2 + L_3)] - T_{e2} \sinh(r_s L_2) + t_a [\sinh(r_s L_2) - \sinh[r_s (L_2 + L_3)]]}{\cosh(r_s L_2) \sinh[r_s (L_2 + L_3)] - \sinh(r_s L_2) \cosh[r_s (L_2 + L_3)]} \] \hspace{1cm} \text{(A6.29)}

\[ T_z = r_s C_{10} \sinh(r_s z) + r_s C_{11} \cosh(r_s z) \] \hspace{1cm} \text{(A6.30)}

For \( L_2 + L_3 \leq z \leq L_2 + L_3 + L_4, \)
Net rate of heat loss by conduction in $z$-direction from volume element $\Delta z$

\[
-\left\{ kA_x \frac{d^2 T_z}{dz^2} \Delta z \right\} + \left\{ h_f' \frac{2 L \Delta z}{kA_x} [T_z - t_a] \right\} = 0
\]

\[
\frac{d^2 T_z}{dz^2} - \frac{2 h_f' L}{kA_x} [T_z - t_a] = 0
\]

Let $r_6 = \sqrt{\frac{2 h_f' L}{kA_x}}$

\[
\frac{d^2 T_z}{dz^2} - \frac{2 h_f' L}{kA_x} [T_z - t_a] = 0
\]

$T_z = C_{12} \text{Cosh}(r_6 z) + C_{13} \text{Sinh}(r_6 z) + t_a$

Boundary condition 1: $z = L_2 + L_3$, $T_z = T_{e2}$,

$T_{e2} = C_{12} \text{Cosh}(r_6 (L_2 + L_3)) + C_{13} \text{Sinh}(r_6 (L_2 + L_3)) + t_a$ ... (A6.31)

Boundary condition 2: $z = L_2 + L_3 + L_4$, $T_z = T_{e^*'}$,

$T_{e^*'} = C_{12} \text{Cosh}(r_6 (L_2 + L_3 + L_4)) + C_{13} \text{Sinh}(r_6 (L_2 + L_3 + L_4)) + t_a$ ... (A6.32)

Equation (A6.31) $\times \text{Cosh}(r_6 (L_2 + L_3 + L_4))$ - Equation (A6.32) $\times \text{Cosh}(r_6 (L_2 + L_3))$,

$T_{e2} \text{Cosh}(r_6 (L_2 + L_3 + L_4)) - T_{e^*'} \text{Cosh}(r_6 (L_2 + L_3))$

$= C_{12} (\text{Sinh}(r_6 (L_2 + L_3)) \text{Cosh}(r_6 (L_2 + L_3 + L_4))$

$- \text{Cosh}(r_6 (L_2 + L_3)) \text{Sinh}(r_6 (L_2 + L_3 + L_4)))$

$+ t_a (\text{Cosh}(r_6 (L_2 + L_3 + L_4)) - \text{Cosh}(r_6 (L_2 + L_3))))$
\[ C_{13} = \{ T_{e2} \cosh[ r_6 ( L_2 + L_3 + L_4 )] - T_{ip} \cosh[ r_6 ( L_2 + L_3 )] \} + t_a \{ \cosh[ r_6 ( L_2 + L_3 + L_4 )] - \cosh[ r_6 ( L_2 + L_3 )] \} \}
\]
\[ / \{ \sinh[ r_6 ( L_2 + L_3 )] \cosh[ r_6 ( L_2 + L_3 + L_4 )] - \cosh[ r_6 ( L_2 + L_3 )] \sinh[ r_6 ( L_2 + L_3 + L_4 )] \} \]
\[ = C_{12} \{ \cosh[ r_6 ( L_2 + L_3 )] \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \sinh[ r_6 ( L_2 + L_3 )] \cosh[ r_6 ( L_2 + L_3 + L_4 )] \} + t_a \{ \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \sinh[ r_6 ( L_2 + L_3 )] \} \]
\[ = C_{12} \{ T_{e2} \cosh[ r_6 ( L_2 + L_3 + L_4 )] - T_{ip} \cosh[ r_6 ( L_2 + L_3 )] \} + t_a \{ \sinh[ r_6 ( L_2 + L_3 )] - \sinh[ r_6 ( L_2 + L_3 + L_4 )] \} \]
\[ / \{ \cosh[ r_6 ( L_2 + L_3 )] \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \sinh[ r_6 ( L_2 + L_3 )] \cosh[ r_6 ( L_2 + L_3 + L_4 )] \} \]
\[ = C_{12} \{ T_{e2} \cosh[ r_6 ( L_2 + L_3 + L_4 )] - T_{ip} \cosh[ r_6 ( L_2 + L_3 )] \} + t_a \{ \sinh[ r_6 ( L_2 + L_3 )] - \sinh[ r_6 ( L_2 + L_3 + L_4 )] \} \]
\[ / \{ \cosh[ r_6 ( L_2 + L_3 )] \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \sinh[ r_6 ( L_2 + L_3 )] \cosh[ r_6 ( L_2 + L_3 + L_4 )] \} \]
\[ \text{...}(A6.33) \]

Equation (A6.31) \times \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \text{Equation (A6.32) \times } \sinh[ r_6 ( L_2 + L_3 )],

\[ T_{e2} \sinh[ r_6 ( L_2 + L_3 + L_4 )] - T_{ip} \sinh[ r_6 ( L_2 + L_3 )] \]
\[ = C_{12} \{ \cosh[ r_6 ( L_2 + L_3 )] \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \sinh[ r_6 ( L_2 + L_3 )] \cosh[ r_6 ( L_2 + L_3 + L_4 )] \} + t_a \{ \sinh[ r_6 ( L_2 + L_3 + L_4 )] - \sinh[ r_6 ( L_2 + L_3 )] \} \]
\[ \text{...}(A6.34) \]

For \( 0 \leq z \leq L_2 / 2, \)

\[ T_z = C_{14} \cosh( r_4 z ) + C_{15} \sinh( r_4 z ) + C_9 \]

Boundary condition 1: \( z = 0, \quad T_z = t_{ip}'' + C_9 \)

\[ t_{ip}'' = C_{14} + C_9 \]
\[ C_{14} = t_{ip}'' - C_9 \]

Boundary condition 2: \( z = L_2 / 2, \quad T_z = T_e, \)

\[ T_e = C_{14} \cosh( t_L / 2 ) + C_{15} \sinh( t_L / 2 ) + C_9 \]
\[ C_{15} = \frac{ (T_e - C_9) - (t_{ip}'' - C_9) \cosh( t_L / 2 ) }{ \sinh( t_L / 2 ) } \]
Heat loss from the system through the edge plate x-sectional area by conduction

For $L_2 / 2 \leq z \leq L_2$,

$$T_z' = r_4 C_7 \text{Sinh}(r_4 z) + r_4 C_8 \text{Cosh}(r_4 z) \quad \text{(A6.35)}$$

For $L_2 \leq z \leq L_2 + L_3$,

$$T_z' = r_3 C_{10} \text{Sinh}(r_3 z) + r_3 C_{11} \text{Cosh}(r_3 z) \quad \text{(A6.36)}$$

For $L_2 + L_3 \leq z \leq L_2 + L_1 + L_4$,

$$T_z' = r_6 C_{12} \text{Sinh}(r_6 z) + r_6 C_{13} \text{Cosh}(r_6 z) \quad \text{(A6.37)}$$

For $0 \leq z \leq L_2 / 2$,

$$T_z' = r_4 C_{14} \text{Sinh}(r_4 z) + r_4 C_{15} \text{Cosh}(r_4 z) \quad \text{(A6.38)}$$

For $z = L_2$, RHS of (A6.35) = RHS of (A6.36),

$$r_4 C_7 \text{Sinh}(r_4 L_2) - r_4 C_8 \text{Cosh}(r_4 L_2) = r_3 C_{10} \text{Sinh}(r_3 L_2) + r_3 C_{11} \text{Cosh}(r_3 L_2) \quad \text{(A6.39)}$$

Since

$$\text{Sinh}\left(\frac{r_4 L_2}{2}\right) \text{Cosh}(r_4 L_2) - \text{Sinh}(r_4 L_2) \text{Cosh}\left(\frac{r_4 L_2}{2}\right) = -\text{Sinh}\left(\frac{r_4 L_2}{2}\right)$$

Therefore

$$C_8 = -\frac{\text{Cosh}(r_4 L_2)}{\text{Sinh}\left(\frac{r_4 L_2}{2}\right)} T_e + \frac{\text{Cosh}\left(\frac{r_4 L_2}{2}\right)}{\text{Sinh}\left(\frac{r_4 L_2}{2}\right)} T_c + \frac{\text{Cosh}(r_4 L_2) - \text{Cosh}\left(\frac{r_4 L_2}{2}\right)}{\text{Sinh}\left(\frac{r_4 L_2}{2}\right)} - C_9 \quad \text{(A6.40)}$$
and

\[ C_9 = -\frac{\sinh (r_5 L_2)}{\sinh \left( \frac{r_4 L_2}{2} \right)} T_e - T_{e1} - \frac{\sinh (r_4 L_2) - \sinh \left( \frac{r_4 L_2}{2} \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} C_9 \]  \hspace{1cm} \text{(A6.41)}

And since

\[ \sinh (r_5 L_2) \cosh [r_5 (L_2 + L_3)] - \cosh (r_5 L_2) \sinh [r_5 (L_2 + L_3)] = -\sinh (r_5 L_3) \]

Therefore

\[ C_{ii} = -\frac{\cosh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} T_{e1} + \frac{\cosh (r_5 L_2)}{\sinh (r_5 L_3)} T_{e2} - \frac{\cosh (r_5 L_2) - \cosh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} t_a \]  \hspace{1cm} \text{(A6.42)}

and

\[ C_{io} = -\frac{\sinh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} T_{e1} - \frac{\sinh (r_5 L_2)}{\sinh (r_5 L_3)} T_{e2} + \frac{\sinh (r_5 L_2) - \sinh [r_5 (L_2 + L_3)]}{\sinh (r_5 L_3)} t_a \]  \hspace{1cm} \text{(A6.43)}

Substituting Equations (A6.40), (A6.41), (A6.42) and (A6.43) into Equation (A6.39) yields
Appendices

\[
\frac{r_4 \sinh^2 \left( r_4 L_2 \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} - \frac{r_4 \cosh^2 \left( r_4 L_2 \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} \right) T_e + \left\{ -r_4 \sinh (r_4 L_2) \right\} C_9
\]

+ \left\{ \frac{\sinh^2 (r_4 L_2) - \sinh (r_4 L_2) \sinh \left( \frac{r_4 L_2}{2} \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} \right\} T_e

+ \left\{ \frac{-r_4}{\sinh \left( \frac{r_4 L_2}{2} \right)} \right\} C_9

= \left\{ \frac{r_5 \sinh (r_5 L_3) \sinh \left[ r_5 (L_2 + L_3) \right]}{\sinh (r_5 L_3)} - \frac{r_5 \cosh (r_5 L_3) \cosh \left[ r_5 (L_2 + L_3) \right]}{\sinh (r_5 L_3)} \right\} T_e

+ \left\{ \frac{-r_5 \sinh^2 (r_5 L_3) + r_5 \cosh^2 (r_5 L_3)}{\sinh (r_5 L_3)} \right\} T_e

+ \left\{ \frac{r_5 \sinh^2 (r_5 L_3) - \sinh (r_5 L_2) \sinh \left[ r_5 (L_2 + L_3) \right]}{\sinh (r_5 L_3)} \right\} t_a

- \frac{r_5}{\sinh (r_5 L_3)} \left\{ 1 - \cosh (r_5 L_3) \right\} t_a

- \frac{r_5 \cosh \left( \frac{r_5 L_2}{2} \right)}{\sinh \left( \frac{r_5 L_3}{2} \right)} \left\{ 1 - \cosh \left( \frac{r_5 L_2}{2} \right) \right\} C_a

Solving the above equation, the following is obtained:

\[
\frac{r_4 \cosh \left( \frac{r_4 L_2}{2} \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} + \frac{r_4 \cosh (r_4 L_3)}{\sinh (r_4 L_3)} \right) T_e = \frac{r_4}{\sinh \left( \frac{r_4 L_2}{2} \right)} T_e

+ \frac{r_5}{\sinh (r_5 L_3)} T_e

- \frac{r_5}{\sinh (r_5 L_3)} \left\{ 1 - \cosh (r_5 L_3) \right\} t_a

- \frac{r_5 \cosh \left( \frac{r_5 L_2}{2} \right)}{\sinh \left( \frac{r_5 L_3}{2} \right)} \left\{ 1 - \cosh \left( \frac{r_5 L_2}{2} \right) \right\} C_a

Let \( C_{16} = \frac{r_4 \cosh \left( \frac{r_4 L_2}{2} \right)}{\sinh \left( \frac{r_4 L_2}{2} \right)} + \frac{r_4 \cosh (r_4 L_3)}{\sinh (r_4 L_3)} \)
Therefore

\[
T_{e1} = \frac{r_4}{C_{16} \sinh \left( \frac{r_4 L_2}{2} \right)} T + \frac{r_5}{C_{16} \sinh (r_5 L_1)} T_{c2} - \frac{r_5}{C_{16} \sinh (r_5 L_3)} \left\{ 1 - \cosh (r_5 L_3) \right\} t_a
\]

\[- \frac{r_4}{C_{16} \sinh \left( \frac{r_4 L_2}{2} \right)} \left\{ 1 - \cosh \left( \frac{r_4 L_2}{2} \right) \right\} C_0
\]

...(A6.44)

For \( z = L_2 + L_3 \), RHS of Equation (A6.36) = RHS of Equation (A6.37).

\[r_5 C_{10} \sinh [r_5 (L_2 + L_3)] + r_5 C_{11} \cosh [r_5 (L_2 + L_3)] = r_6 C_{13} \sinh [r_6 (L_2 + L_3)] \]
\[+ r_6 C_{13} \cosh [r_6 (L_2 + L_3)] \]

...(A6.45)

Since

\[\sinh [r_6 (L_2 + L_3)] \cosh [r_6 (L_2 + L_3) + L_4] \]
\[- \sinh [r_6 (L_2 + L_3 + L_4)] \cosh [r_6 (L_2 + L_3)] = - \sinh (r_6 L_4)
\]

Therefore

\[C_{13} = \frac{\cosh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} T_{c2} + \frac{\cosh [r_6 (L_2 + L_3) + L_4]}{\sinh (r_6 L_4)} t_{op}, \]

\[- \frac{\cosh [r_6 (L_2 + L_3)] - \cosh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} t_a
\]

...(A6.46)

and

\[C_{12} = \frac{\sinh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} T_{c2} - \frac{\sinh [r_6 (L_2 + L_3)]}{\sinh (r_6 L_4)} t_{op}, \]

\[+ \frac{\sinh [r_6 (L_2 + L_3)] - \sinh [r_6 (L_2 + L_3 + L_4)]}{\sinh (r_6 L_4)} t_a
\]

...(A6.47)

Substituting Equation (A6.42), (A6.43), (A6.46) and (A6.47) into Equation (A6.45) yields
Let

\[ C_{17} = \frac{r_5 \cosh(r_5 L_3)}{\sinh(r_5 L_3)} + \frac{r_6 \cosh(r_6 L_4)}{\sinh(r_6 L_4)} \]  \quad \text{(A6.49)}

Therefore

\[ T_{c2} = \frac{r_5}{C_{17} \sinh(r_5 L_3)} T_{c1} + \frac{r_6}{C_{17} \sinh(r_6 L_4)} t_{vp} \]

\[ - \left\{ \frac{r_5}{C_{17} \sinh(r_5 L_3)} [1 - \cosh(r_5 L_3)] + \frac{r_6}{C_{17} \sinh(r_6 L_4)} [1 - \cosh(r_6 L_4)] \right\} t_a \]  \quad \text{(A6.50)}

Substituting Equation (A6.50) into Equation (A6.44) yields
\[ T_{el} = \frac{r_4}{C_{16} \text{Sinh} \left( \frac{r_4 L_2}{2} \right)} T_e - \frac{r_5}{C_{16} \text{Sinh} (r_5 L_3)} \left[ 1 - \text{Cosh} (r_5 L_3) \right] T_a \\
- \frac{r_4}{C_{16} \text{Sinh} \left( \frac{r_4 L_2}{2} \right)} \left[ 1 - \text{Cosh} \left( \frac{r_4 L_2}{2} \right) \right] C_9 \\
+ \frac{r_5}{C_{16} \text{Sinh} (r_5 L_3)} \left\{ \frac{r_3}{C_{17} \text{Sinh} (r_3 L_3)} T_{el} + \frac{r_6}{C_{17} \text{Sinh} (r_6 L_4)} t_{up} \right\} \\
- \left\{ \frac{r_5}{C_{17} \text{Sinh} (r_5 L_3)} \left[ 1 - \text{Cosh} (r_5 L_3) \right] + \frac{r_6}{C_{17} \text{Sinh} (r_6 L_4)} \left[ 1 - \text{Cosh} (r_6 L_4) \right] \right\} t_a \\
\left\{ 1 - \frac{r_5^2}{C_{16} C_{17} \text{Sinh}^2 (r_5 L_3)} \right\} T_{el} = \frac{r_4}{C_{16} \text{Sinh} \left( \frac{r_4 L_2}{2} \right)} T_e \\
- \frac{r_4}{C_{16} \text{Sinh} \left( \frac{r_4 L_2}{2} \right)} \left[ 1 - \text{Cosh} \left( \frac{r_4 L_2}{2} \right) \right] C_9 + \frac{r_6}{C_{16} C_{17} \text{Sinh} (r_5 L_3) \text{Sinh} (r_6 L_4)} t_{up} \\
- \frac{r_5}{C_{16} \text{Sinh} (r_5 L_3)} \left\{ \left[ 1 - \text{Cosh} (r_5 L_3) \right] + \frac{r_5}{C_{17} \text{Sinh} (r_5 L_3)} \left[ 1 - \text{Cosh} (r_5 L_3) \right] \right\} \\
+ \frac{r_6}{C_{17} \text{Sinh} (r_6 L_4)} \left[ 1 - \text{Cosh} (r_6 L_4) \right] \right\} t_a \\
\right\} t_a \\
\text{(A6.51)} \\
\text{Let} \\
\text{C}_{18} = 1 - \frac{r_5^2}{C_{16} C_{17} \text{Sinh}^2 (r_5 L_3)} \\
\text{(A6.52)} \\
\text{Therefore,}
\[ T_{e1} = \frac{r_4}{C_{16} C_{18} \sinh \left( \frac{r_4 L_2}{2} \right)} T_e - \frac{r_4}{C_{16} C_{18} \sinh \left( \frac{r_4 L_2}{2} \right)} \left[ 1 - \cosh \left( \frac{r_4 L_2}{2} \right) \right] C_9 \]

\[ + \frac{r_4 r_6}{C_{16} C_{17} C_{18} \sinh (r_5 L_3) \sinh (r_6 L_4)} t_{up} \]

\[ - \frac{r_5}{C_{16} C_{18} \sinh (r_5 L_3)} \left\{ [1 - \cosh (r_5 L_3)] + \frac{r_5}{C_{17} \sinh (r_5 L_3)} [1 - \cosh (r_5 L_3)] \right\} t_a \]

\[ + \frac{r_6}{C_{17} \sinh (r_6 L_4)} [1 - \cosh (r_6 L_4)] \right\} t_a \]

\[ \text{Therefore,} \]

\[ Q_{\text{extend-edge}} = kA_4 r_4 C_{15} - kA_4 r_4 \left[ C_{14} \sinh (r_4 L_2) + C_{15} \cosh (r_4 L_2) \right] \]
A5. Experimental uncertainties

There are many sources of uncertainties, and in the observer's mind their effects are often combined and related in the notion of experimental uncertainty, conveniently linked as the reason for discrepancies between observed and expected results. The two classes of uncertainties in an experimental measurement are due to bias (systematic) error and precision (random) error. These are caused by fluctuations in temperatures, irradiance level, pressure (mass flow rate) and the way which the measurements were taken.

Suppose a set of measurements is made and the uncertainty in each measurement may be expressed with the same odds. These measurements are then used to calculate some desired result of the experiments. The uncertainty in the calculated result is estimated on the basis of the uncertainties in the primary measurements. The results $R$ is a given function of the independent variables $x_1, x_2, x_3, ..., x_n$ (Holman, 1984). Thus,

$$ R = R(x_1, x_2, x_3, ..., x_n) \quad \ldots (A5.1) $$

If $w_R$ is the uncertainty in the result and $w_1, w_2, ..., w_n$ are the uncertainties in the independent variables and if the uncertainties in the independent variables are all given with the same odds, then the uncertainty in the result having these odds will be (Holman, 1984):

$$ w_R = \left[ \left( \frac{\partial R}{\partial x_1} \right)^2 + \left( \frac{\partial R}{\partial x_2} \right)^2 + \cdots + \left( \frac{\partial R}{\partial x_n} \right)^2 \right]^{1/2} \quad \ldots (A5.2) $$

Suppose a set of data is collected in the variables $x_1, x_2, x_3, ..., x_n$ and a result calculated. At the same time one may perturb the variables by $Dx_1, Dx_2$, and so on, and calculate new results. The following would then be derived (Holman, 1984):

$$ R(x_1) = R(x_1, x_2, ..., x_n) \quad \ldots (A5.3) $$

$$ R(x_1 + Dx_1) = R(x_1 + Dx_1, x_2, ..., x_n) \quad \ldots (A5.4) $$

and,

$$ R(x_2) = R(x_1, x_2, ..., x_n) \quad \ldots (A5.5) $$
\[ R(x_2 + \Delta x_2) = R(x_1, x_2 + \Delta x_2, \ldots, x_n) \]  \hspace{1cm} \text{(A5.6)}

For small value of \( \Delta x \) the partial derivatives can be well approximated by (Holman, 1984):

\[
\frac{\partial R}{\partial x_1} \approx \frac{R(x_1 + \Delta x_1) - R(x_1)}{\Delta x_1} \hspace{1cm} \text{(A5.7)}
\]

\[
\frac{\partial R}{\partial x_2} \approx \frac{R(x_2 + \Delta x_2) - R(x_2)}{\Delta x_2} \hspace{1cm} \text{(A5.8)}
\]

The estimation of efficiency uncertainty was carried out by considering the uncertainties in the following variables:

- simulated irradiance
- ambient-inlet temperature difference
- mass flow rate of air in collector
- area of collector surface

The overall uncertainty of a test was then derived as follows:

\[
\delta m = \rho q \sqrt{\sum_{i=1}^{n} \frac{A_i v_i}{\sqrt{\rho}}} \hspace{1cm} \text{(A5.11)}
\]

\[
\frac{\delta m}{\delta A_i} = v_i \sqrt{\frac{\rho}{\rho}} \hspace{1cm} \text{(A5.12)}
\]

Likewise,
\[
\frac{\delta m}{\delta A_v} = v, \sqrt{2\rho} \quad \text{...(A5.13)}
\]

\[
\frac{\delta m}{\delta v_j} = A_j \sqrt{2\rho} \quad \text{...(A5.14)}
\]

Likewise,

\[
\frac{\delta m}{\delta v_n} = A_n \sqrt{2\rho} \quad \text{...(A5.15)}
\]

\[
\frac{\delta m}{\delta \rho} = \frac{1}{\sqrt{2\rho}} \sum_{r=1}^{n} (A_r v_r) \quad \text{...(A5.16)}
\]

\[
m = \left\{ \left[ \frac{\partial m}{\partial A_1} W_{A_1} \right]^2 + \left( \frac{\partial m}{\partial A_1} W_{A_1} \right)^2 + \cdots + \left( \frac{\partial m}{\partial A_n} W_{A_n} \right)^2 \right\}^{1/2}
\]

\[
+ \left\{ \left[ \frac{\partial m}{\partial v_1} W_{v_1} \right]^2 + \left( \frac{\partial m}{\partial v_2} W_{v_2} \right)^2 + \cdots + \left( \frac{\partial m}{\partial v_n} W_{v_n} \right)^2 \right\}^{1/2}
\]

\[
+ \left\{ \frac{\partial m}{\partial \rho} \sum_{r=1}^{n} (A_r v_r) \cdot W_{\rho} \right\}^{1/2}
\]

\[
\]

\[
= \left\{ \left( \sqrt{2\rho} v_1 W_{A_1} \right)^2 + \left( \sqrt{2\rho} v_2 W_{A_1} \right)^2 + \cdots + \left( \sqrt{2\rho} v_n W_{A_1} \right)^2 \right\}^{1/2}
\]

\[
+ \left\{ \left( \sqrt{2\rho} A_1 W_{v_1} \right)^2 + \left( \sqrt{2\rho} A_2 W_{v_2} \right)^2 + \cdots + \left( \sqrt{2\rho} A_n W_{v_n} \right)^2 \right\}^{1/2}
\]

\[
+ \left\{ \frac{\sqrt{2\rho}}{2} \sum_{r=1}^{n} (A_r v_r) \cdot W_{\rho} \right\}^{1/2}
\]

\[
= \left\{ \left( 2\rho v_1 W_{A_1} \right)^2 + \left( 2\rho v_2 W_{A_1} \right)^2 + \cdots + \left( 2\rho v_n W_{A_1} \right)^2 \right\}^{1/2}
\]

\[
+ \left\{ \left( 2\rho A_1 W_{v_1} \right)^2 + \left( 2\rho A_2 W_{v_2} \right)^2 + \cdots + \left( 2\rho A_n W_{v_n} \right)^2 \right\}^{1/2}
\]

\[
+ \frac{1}{2\rho} \left[ \sum_{r=1}^{n} (A_r v_r) \right] \cdot W_{\rho}^{1/2}
\]

\[
= \left\{ 2\rho \sum_{r=1}^{n} (v_r W_{A_r})^2 + 2\rho \sum_{r=1}^{n} (A_r W_{v_r})^2 + \frac{1}{2\rho} \left[ \sum_{r=1}^{n} (A_r v_r) \right] \cdot W_{\rho}^{1/2} \right\}^{1/2}
\]

\[
\quad \text{...(A5.17)}
\]

The following equation is how efficiency was calculated from the laboratory measurement:

\[
\eta = \frac{mC_p \Delta t}{lJ} \quad \text{...(A5.19)}
\]
Let $W_\eta$ be the uncertainty in $\eta$ and $W_m$, $W_{\Delta t}$, $W_I$ and $W_A$ be the uncertainties in the independent variables $m$, $\Delta t$, $I$ and $A$.

If the uncertainties in the independent variables are all given with the same odds (probability), then the uncertainty in the result having these odds is:

\[
W_\eta = \left[ \left( \frac{\delta \eta}{\delta m} \frac{W_m}{m} \right)^2 + \left( \frac{\delta \eta}{\delta \Delta t} \frac{W_{\Delta t}}{\Delta t} \right)^2 + \left( \frac{\delta \eta}{\delta I} \frac{W_I}{I} \right)^2 + \left( \frac{\delta \eta}{\delta A} \frac{W_A}{A} \right)^2 \right]^{1/2}
\]

...(A5.24)

\[
W_\eta \cdot \eta = \left[ \left( \frac{\eta}{m} \right)^2 + \left( \frac{\eta}{\Delta t} \right)^2 + \left( \frac{\eta}{I} \right)^2 + \left( \frac{\eta}{A} \right)^2 \right]^{1/2}
\]

...(A5.25)

\[
\frac{W_\eta}{\eta} = \left[ \left( \frac{W_m}{m} \right)^2 + \left( \frac{W_{\Delta t}}{\Delta t} \right)^2 + \left( \frac{W_I}{I} \right)^2 + \left( \frac{W_A}{A} \right)^2 \right]^{1/2}
\]

...(A5.26)
References

A6. Computer program listings

A6.1. Program listing for the proposed air heating solar collector (TRNSYS)

SUBROUTINE TYPE59(TIME, XIN, OUT, DTDT, PAR, INFO, ICNTRL, *)

CHARACTER*3 YCHECK(11), OCHECK(26)

DOUBLE PRECISION XIN, OUT

DIMENSION XIN(21), PAR(31), OUT(26), INFO(15)

PARAMETERS:

PR1 = Prandtl number of transport fluid [-]
CP = specific heat capacity of transport fluid [J/Kg*C]
RHO = density of transport fluid [Kg/m3]
L = length of the collector [m]
W = width of the collector [m]
D = gap between layers in cover system [m]
AX = x-sectional area of flow channel [m2]
ETAC = emissivity of layers in cover system [-]
ETA3 = emissivity of absorber surface [-]
ETA4 = emissivity of the rear surface [-]
K2 = rear insulation conductivity [W/mC]
D2 = rear insulation thickness [m]
FN = perimeter of flow channel x-section [m]
AE = edge plate area (one side only) [m2]
KE = edge insulation conductivity [W/mC]
DE = edge plate thickness [m]
K = absorber-cover area ratio [-]
AR = aspect ratio of cover system compartments [-]
ALPHA = absorber's absorption [-]
SLOPE = collector's tilt angle [degree]
XKL = extinction coefficient of cover layer thickness product [-]
RHOG = ground reflectance [-]
AR1 = flow channel's aspect ratio [-]
LCR = cube root of the building volume [m]
INOUT = 0 for indoor; 1 for outdoor [-]
TABS = tau_alpha effective [-]
TAUIR = transmissivity of cover to IR radiation [-]
KS = terrain dependent variable (refer to CIBSE guide) [-]
Z = height (mid-height of building) [m]
AA = terrain dependent variable (refer to CIBSE guide) [-]
MD = mean channel depth [m]

INPUTs:

V10 = wind speed measured at 10 metres above ground
      (taken from meteorological data) [m/s]
VPD = wind speed across collector rear surface [m/s]
TA = top ambient temperature [*C]
TAD = rear ambient temperature [*C]
TI = inlet temperature [*C]
M = transport fluid's mass flow rate
I = total horizontal radiation [W/m2]
IT = total incident radiation on the collector [W/m2]
ID = horizontal diffuse radiation [W/m2]
THETA = incident angle of radiation [degree]
TDP = dew point temperature of outdoor environment [*C]

VP = wind speed across cover outer surface [m/s]
T1 = temperature of the top surface of the cover system [*C]
T1A = temperature of the middle layer of the cover system [*C]
Appendices

- \( T_2 \) = temperature of the bottom surface of the cover system [\(^\circ\)C]
- \( K_{COVER} \) = equivalent conductance of the cover system, including the effect of convection and radiation heat transfer within the layers [W/m\(^2\)\(^\circ\)C]
- \( \text{RHOD} \) = -1 (for SUBROUTINE TALF) [-]
- \( \text{SLOPE} \) = collector slope [\(^\circ\)]
- \( \text{EFFSKY} \) = effective incidence angle for sky diffuse radiation [degree]
- \( \text{EFFGND} \) = effective incidence angle for ground reflected radiation [degree]
- \( \text{COSSLP} \) = \( \cos(\text{SLOPE} \cdot \text{RDCONV}) \) [-1]
- \( \text{RDCONV} \) = degree-radian conversion factor [-]
- \( \text{FSKY} \) = \( (1 + \text{COSSLP}) / 2 \) [-]
- \( \text{FGND} \) = \( (1 - \text{COSSLP}) / 2 \) [-]
- \( \text{GDSKY} \) = \( \text{FSKY} \cdot \text{ID} \) [W/m\(^2\)l
- \( \text{GDGND} \) = \( \text{RHOG} \cdot \text{FGND} \cdot \text{I} \) [W/m\(^2\)l
- \( \tau_{\text{sky}} \) = transmission factor for sky [W/m\(^2\)\(^\circ\)C]
- \( \tau_{\text{gnd}} \) = transmission factor for ground [W/m\(^2\)\(^\circ\)C]
- \( \text{LD} \) = \( (L + W) / 2 \) [m]
- \( \text{VISF} \) = kinematic viscosity of transport fluid (based on mean fluid temperature) [m\(^2\)/s]
- \( \text{KF} \) = transport fluid conductivity (based on mean fluid temperature) [W/m\(^\circ\)C]
- \( \text{V} \) = transport fluid velocity [m/s]
- \( \text{DN} \) = hydraulic diameter of flow channel [m]
- \( \text{RE} \) = Reynolds number of transport fluid [-]
- \( \text{NU} \) = Nusselt number of transport fluid [-1]
- \( H_2 \) = convection heat transfer coefficient between transport fluid and surface 2 (cover inner surface) [W/m\(^2\)\(^\circ\)C]
- \( H_3 \) = convection heat transfer coefficient between transport fluid and surface 3 (absorber) [W/m\(^2\)\(^\circ\)C]
- \( \text{KA1} \) = transport fluid conductivity (based on mean surface temperature) [W/m\(^\circ\)C]
- \( \text{VIS1} \) = kinematic viscosity of transport fluid (based on mean surface temperature) [m\(^2\)/s]
- \( \text{BETA1} \) = linear expansion coefficient of transport air (based on mean surface temperature) [\(^*/\)C]
- \( \text{RA1} \) = Rayleigh number of transport fluid [-]
- \( \text{HR23} \) = radiation heat transfer coefficient between the interior surface and the absorber [W/m\(^2\)\(^\circ\)C]
- \( \text{MIGNIG} \) = time from mid-night [h]
- \( \text{TS} \) = sky temperature [\(^\circ\)C]
- \( \text{HR1} \) = radiation heat transfer coefficient between cover outer surface and outdoor environment [W/m\(^2\)\(^\circ\)C]
- \( \text{SBC} \) = Stefan-Boltzmann constant [W/m\(^2\)\(^\circ\)C4]
- \( = 5.67 \times 10^{-8} \)
- \( \text{UE} \) = overall heat transfer coefficient between the edge and outdoor environment [W/m\(^2\)\(^\circ\)C]
- \( \text{H4} \) = convection heat transfer between rear surface and indoor environment [W/m\(^2\)\(^\circ\)C]
- \( \text{UT} \) = overall heat transfer coefficient between the cover (from inner cover surface) and outdoor environment [W/m\(^2\)\(^\circ\)C]
- \( \text{UTD} \) = combined effect of \( \text{UT} \) & \( \text{UE} \) [W/m\(^2\)\(^\circ\)C]
- \( \text{UB} \) = overall heat transfer coefficient between absorber and indoor environment [W/m\(^2\)\(^\circ\)C]
- \( \text{TFD} \) = mean transport fluid temperature [\(^\circ\)C]
- \( \text{TO} \) = outlet temperature [\(^\circ\)C]
- \( \text{UL} \) = overall heat loss coefficient [W/m\(^2\)\(^\circ\)C]
- \( \text{FD} \) = collector efficiency factor [-]
- \( \text{FR} \) = collector heat removal factor [-]
- \( \text{TASTAR} \) = equivalent ambient temperature to HWC equation [\(^\circ\)C]

DOUBLE PRECISION TFDI, TII, T1I, T3I, T4I, MEANT, TDP, TS, TSI, T2S, T3S, T4S, TI, TA, TAD
DOUBLE PRECISION TFD, T1A, T2, T3, T4, TFDIII, TFDII, T1III, T1II, T2III, T2II, T3III, T3II, T4III, T4II, T14, T15, T214, T215, T314, T315, T414, T415

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DOUBLE PRECISION HC11A, HR11A, HC1A2, HR1A2
DOUBLE PRECISION KCOVER, AR
DOUBLE PRECISION LD, VISF, KF, V, DH, RE, NU, NU1, NU2, NU3, NU4, NU5, GG
& NU90, NU60
&, VIS1, KA1, BETAl
DOUBLE PRECISION H2, H3, HR23, Hl, HR1
DOUBLE PRECISION UE, H4, HR4, UT, UTD, UB, UL
DOUBLE PRECISION TO, QU, I, IT, ID, M, VP, VPD
DOUBLE PRECISION AA, Z, KS, HR3S, TAEF, H23, MD
REAL L, W, D, AX
REAL ETAC, ETA3, ETA4, K2, D2, PW, AE, KE, DE
REAL K, MIDNIG, EFFSKY, EFFGND, COSSLP, THETA
REAL FSky, FGND, GDSKY, GDGND, TASKY, TAGND, TABM, RHOD, RHOG
REAL DIFFER
INTEGER FLAG

C Declare iteration counter
INTEGER LOOP

PARAMETER (G=9.81, SBC=5.67E-8, RDCONV=0.017453)

C Set INFOs
INFO(3)=11
INFO(4)=31
INFO(6)=26
INFO(9)=1

CALL TYPECK(1, INFO, 11, 31, 0)

DATA YCHECK/'VEl', 'VEl', 'TEl', 'TEl', 'TEl', 'MF1', 'IRl', 'IRl', 'IRl',
& 'DG1', 'TE1'/
DATA OCHECK/'TEl', 'TEl', 'TEl', 'TEl', 'TEl', 'HT1', 'HT1', 'HT1', 'HT1',
& 'HT1', 'HT1', 'HT1', 'HT1', 'HT1', 'HT1', 'HT1', 'HT1', 'PW1', 'MF1', 'HT11',
& 'DMl1', 'DMl', 'TEl', 'PW1', 'PW1'/
CALL RCHECK(INFO, YCHECK, OCHECK)

C Set PARameters (with unit conversion)
PRI=PAR(1)
CP=1000*PAR(2)
RHO=PAR(3)
L=PAR(4)
W=PAR(5)
D=PAR(6)
AX=PAR(7)
ETAC=PAR(8)
ETA3=PAR(9)
ETA4=PAR(10)
K2=0.277778*PAR(11)
D2=PAR(12)
Pw=PAR(13)
AE=PAR(14)
KE=0.277778*PAR(15)
DE=PAR(16)
K=PAR(17)
AR=PAR(18)
ALPHA=PAR(19)
SLOPE=PAR(20)
XXL=PAR(21)
RHOG=PAR(22)
AR1=PAR(23)
LCR=PAR(24)
INOUT=PAR(25)
TAEF=PAR(26)
TAUIR=PAR(27)
KS=PAR(28)
Z=PAR(29)
AA=PAR(30)
MD=PAR(31)
Appendices

C Set INPUTs (with unit conversion)
V10=XIN(1)
VPD=XIN(2)
TA=XIN(3)
TAD=XIN(4)
TI=XIN(5)
M=2.77778E-4*XIN(6)
I=0.277778*XIN(7)
IT=0.277778*XIN(8)
ID=0.277778*XIN(9)
THETA=XIN(10)
TDP=XIN(11)

C Set initial guess values
TFDI=20
T1I=14
T2I=23
T3I=48
T4I=19

10 LOOP=LOOP+1

* Calculate cover thermal conductance *

C Assume temperature of middle layer is the mean of surfaces 1 & 2:
TIA=(T1I+T2I)/2

C Calculate properties between 1 & la:
CALL COVER(T1I, TIA, ETAC, AR, D, SLOPE, HC11A, HR11A)

C Calculate properties between la & 2:
CALL COVER(TIA, T2I, ETAC, AR, D, SLOPE, HC1A2, HR1A2)

C Calculate overall equivalent conductance of the cover:
KCOVER=1/(1/(HC11A+HR11A)+1/(HC1A2+HR1A2))

* Calculate TAU_ALP_e *

C For indoor condition, TAEF is as defined at the beginning of
the program [=PAR(26)]
COSSLP=COS(SLOPE*RDCONV)
IF (INOUT EQ. 0) THEN
GOTO 15
ELSE
RHOD=-1
EFFSKY=59.68-0.1388*SLOPE+0.001497*SLOPE*SLOPE
EPFGND=30.0-0.5788*SLOPE+0.002693*SLOPE*SLOPE
FSKY=(1.0+COSSLP)/2
FGND=(1.0-COSSLP)/2
GDSKY=FSKY*ID
GDGND=RHOG*FGND*I
TASKY=TALF(3, EFFSKY, XKL, 1.586, ALPHA, RHOD)
TAGND=TALF(3, EPFGND, XKL, 1.586, ALPHA, RHOD)
TABM=TALF(3, THETA, XKL, 1.586, ALPHA, RHOD)
IF (IT EQ. 0) THEN
TAEF=0
ELSE
TAEF=(TABM*(GDSKY-GDGND)
&+TASKY*GDSKY+TAGND*GDGND)/IT
IF (TAEF .LT. 0) TAEF=0
END IF
END IF
Appendices

- Heat transfer coefficients in the collector system -

C Characteristic length:
15 \[ LD = \frac{(L+W)}{2} \]

C Kinematic viscosity of channel air:
\[ V\text{ISF} = 9 \times 10^{-10} (\text{TFDI}+273)^{1.72} \]

C Calculate fluid conductivity:
\[ K\text{F} = 0.000206 (\text{TFDI}+273)^{0.85} \]

C Mean channel fluid velocity:
\[ V = \frac{M}{A \cdot X / \text{RHO}} \]

C Hydraulic diameter of the channel:
\[ D\text{H} = 4 \cdot A \cdot X / \text{PW} \]

C Reynolds number:
\[ RE = \frac{V \cdot D\text{H}}{V\text{ISF}} \]

C Nusselt number (whether LAMINAR or TURBULENT):
\[
\text{IF} \ (M \ \leq \ 0) \ \text{GOTO} \ 200 \\
\text{IF} \ (RE \ \geq 2300) \ \text{THEN} \\
\quad NU = 0.0158 \cdot RE^{0.8} \\
\quad N = 0.5 \\
\text{ELSE} \\
\quad NU = 8.235 \\
\quad N = 0 \\
\text{ENDIF} \\
\]

C Convection heat transfer coefficients in the channel:
\[ H2 = NU \cdot K\text{F} / D\text{H} \cdot (1 + (D\text{H} / L)^{0.7}) \cdot (TFDI+273)^{N} \] 
\[ H3 = NU \cdot K\text{F} / D\text{H} \cdot (1 + (D\text{H} / L)^{0.7}) \cdot (T3+273)^{N} \]

200 MEANT1 = (T2I + T3I) / 2
\[ K\text{A1} = 0.000206 \cdot (\text{MEANT1}+273)^{0.85} \]
\[ V\text{IS1} = 9 \times 10^{-10} \cdot (\text{MEANT1}+273)^{1.72} \]
\[ \text{BETA1} = \frac{1}{(\text{MEANT1}+273)} \]
\[ \text{RA1} = G \cdot \text{BETA1} \cdot \text{ABS} (T3I-T2I) \cdot MD^{3} \cdot \text{PR1} / \text{VIS1}^{2} \]

IF (SLOPE \ \leq \ 75) \ \text{THEN} \\
\quad NU1 = 1-1708 / RA1 / \text{COSSLP} \\
\quad \text{IF} \ (NU1 \ \lt \ 0) \ NU1 = 0 \\
\quad NU2 = (RA1 \cdot \text{COSSLP} / 5830)^{0.3333333} - 1 \\
\quad \text{IF} \ (NU2 \ \lt \ 0) \ NU2 = 0 \\
\quad NU = 1.44 \cdot (1-1708 \cdot (\text{SIN} \cdot 1.8 \cdot \text{RDCONV} \cdot \text{SLOPE}))^{1.6} / \text{RA1} / \text{COSSLP} \cdot NU1 + NU2 \\
\text{ELSE} \\
\quad NU1 = 0.0605 \cdot RA1^{(1/3)} \\
\quad NU2 = (1 + (0.104 \cdot RA1^{0.293} / (1+ (6310 / RA1)^{1.36}))^{3})^{(1/3)} \\
\quad NU3 = 0.242 \cdot (RA1 / \text{AR1})^{0.272} \\
\quad NU90 = NU1 \\
\quad \text{IF} \ (NU2 \ \lt \ NU90) \ NU90 = NU2 \\
\quad \text{IF} \ (NU3 \ \lt \ NU90) \ NU90 = NU3 \\
\quad GG = 0.5 / (1+ (RA1/3160)^{20.6})^{0.1} \\
\quad NU4 = (1+ (0.0936 \cdot RA1^{0.314} / (1+GG))^{7})^{(1/7)} \\
\quad NU5 = (0.104 + 0.175 / \text{AR1}) \cdot RA1^{0.283} \\
\quad NU60 = NU4 \\
\quad \text{IF} \ (NU5 \ \lt \ NU60) \ NU60 = NU5 \\
\quad NU = ((90 - \text{SLOPE}) \cdot NU60 + (\text{SLOPE} - 60) \cdot NU90) / 30 \\
\text{ENDIF} \\

C The followings allow prediction of TFD to be made
\[
H2 = NU \cdot K\text{A1} / MD \cdot 2 \\
H3 = NU \cdot K\text{A1} / MD \cdot 2 \]
\[ H_{23} = \frac{\text{NU} \times K_1}{\text{MD}} \]

\[ C \quad \text{Radiation heat transfer coefficient between absorber and cover inner surface:} \]

\[ H_{R23} = S_{\text{BC}} \times \left( \frac{(T_{2I}+273)^2 + (T_{3I}+273)^2}{T_{2I}+T_{3I}+546} \right) \times (l/\eta_{\text{AC}}+i/\eta_{\text{A3}}-1) \]

\[ C \quad \text{Determine sky temperature. For indoor condition, } ts=ta \]

\[
\text{IF (INOUT .EQ. 0) THEN} \\
\text{TS} = TA \\
\text{ELSE} \\
\text{TS1} = TA + 273 \\
\text{TS2} = 0.0056 \times TDP \\
\text{TS3} = 0.000073 \times TDP \times TDP \\
\text{TS4} = 0.013 \times \cos(15 \times \text{RDCONV} \times \text{MIDNIG}) \\
\text{TS} = TS1 \times (0.711 + TS2 + TS3 + TS4) ** 0.25 - 273 \\
\text{END IF} \]

\[ C \quad H_l \ & HR_1: \]

\[
\text{IF (INOUT .EQ. 0) THEN} \\
V_P = V_{10} \\
\text{IF (VP .EQ. 0) THEN} \\
H_l = 1.42 \times (\text{ABS}(T_{II}-TA) / LD) ** (0.25) \\
\text{ELSE} \\
H_l = 5.7 + 3.8 \times VP \\
\text{END IF} \\
\text{ELSE} \\
V_P = V_{10} \times K_S \times Z^{** AA} \\
H_l = 8.5 \times VP ** 0.6 / LCR ** (0.4) \\
\text{IF (H_l \ .LE. 5) H_l = 5} \\
\text{END IF} \\
\text{IF (TII \ .EQ. TA) THEN} \\
HR_1 = 4.2 \\
\text{ELSE} \\
HR_1 = \text{ABS}(ETAC \times S_{\text{BC}} \times ((T_{II}+273)^2 + (TS+273)^2) \times (T_{II}-TS) / (T_{II}-TA)) \\
\text{END IF} \]

\[ C \quad HR_3S: \]

\[
\text{IF (T3 .EQ. TA) THEN} \\
HR_3S = 2.0 \\
\text{ELSE} \\
HR_3S = \text{ABS}(\text{TAUR} \times \eta_{\text{A3}} \times S_{\text{BC}} \times ((T_{3I}+273)^4 - (TS+273)^4) / (T_{3I}-TA)) \\
\text{END IF} \]

\[ C \quad UE: \]

\[
UE = 1 / (0.06 + DE / KE) \\
\]

\[ C \quad H_4 \ & HR_4: \]

\[
\text{IF (VPD .EQ. 0) THEN} \\
H_4 = 1.42 \times (\text{ABS}(T_{4I}-TAD) / LD) ** 0.25 \\
\text{ELSE} \\
H_4 = 5.7 + 3.8 \times V_{PD} \\
\text{END IF} \\
\text{HR_4 = ETA_4 \times S_{BC} \times ((T_{4I}+273)^2 + (TAD+273)^2) \times (T_{4I}+TAD+546)} \\
\]

\[ C \quad UT \ & UTD: \]

\[
UT = 1 / (1 / K_{COVER} + 1 / (H_l+HR_1)) \\
UTD = UT + 2 \times (UE \times AE) / L / W \\
\]

\[ C \quad UB: \]

\[
UB = 1 / (D_2 / K_2 + 1 / (H_4 + HR_4)) \\
\]

**Calculate temperature based on HTCs**

**Define P's:**

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\[ P_1 = K \cdot \text{HR23} / (\text{UTD} + H_2 + K \cdot \text{HR23}) - UB / K \cdot \text{HR23} - \text{HR3S} / \text{HR23} - H_3 / \text{HR23} - 1 \]
\[ P_2 = \text{UTD} / (\text{UTD} + H_2 + K \cdot \text{HR23}) + \text{HR3S} / \text{HR23} \]
\[ P_3 = H_3 / \text{HR23} + H_2 / (\text{UTD} + H_2 + K \cdot \text{HR23}) \]
\[ P_4 = H_2 / (\text{UTD} + H_2 + K \cdot \text{HR23}) - K \cdot \text{HR23} / P_1 / (\text{UTD} + H_2 + K \cdot \text{HR23}) \cdot P_1 - 1 \]
\[ P_5 = \text{UTD} / (\text{UTD} + H_2 + K \cdot \text{HR23}) - K \cdot \text{HR23} / P_1 / (\text{UTD} + H_2 + K \cdot \text{HR23}) \cdot P_2 \]
\[ P_6 = H_2 \cdot P_4 - K \cdot H_3 \cdot (P_3 / P_1 + 1) \]
\[ P_7 = H_2 \cdot P_5 - K \cdot H_3 / P_1 \cdot P_2 \]

**C** Overall heat loss coefficient
\[ U_L = P_6 \cdot P_3 / P_3 \]

**C** Collector efficiency factor
\[ F_D = -P_3 / P_1 \]

\[ F_D = K \cdot (H_3 \cdot \text{UTD} + H_3 \cdot H_2 + K \cdot H_3 \cdot \text{HR23} + H_2 \cdot \text{HR23}) \]
\[ / (\text{UTD} + H_2 + K \cdot \text{HR23}) \cdot (UB + K \cdot \text{HR3S} + K \cdot H_3) + K \cdot \text{HR23} / (\text{UTD} + H_2) \]

**C** Collector heat removal

**C** No flow!
\[ T_{265} = \text{UTD} / P_1 \]

**C** Find out the fluid outlet temperature, \( T_O \):
\[ T_{265} = \text{UTD} - UB / P_1 \]

**C** Calculate mean fluid temperature
\[ P_9 = M \cdot \text{CP} / P_6 / \text{W} / \text{L} \]

\[ T_{265} = T_{265} + 1 \cdot P_9 \cdot (\text{EXP}(1 / P_9) - 1) \]

\[ + TI \cdot (\text{EXP}(1 / P_9) - 1) \cdot P_9 \]

\[ T_{265} = (T_{265} + IT \cdot \text{TAEF} / UL) / (1 - P_9 \cdot (\text{EXP}(1 / P_9) - 1) \cdot P_9 \]

\[ + (1 \cdot P_9 \cdot (\text{EXP}(1 / P_9) - 1)) \]

\[ + (1 - P_9 \cdot (\text{EXP}(1 / P_9) - 1)) \]

\[ + TI \cdot (\text{EXP}(1 / P_9) - 1) \cdot P_9 \]

**C** T2:
\[ (IT \cdot \text{EQ. 0}) \]

\[ T_2 = -UB / P_1 / (\text{UTD} + H_2 + K \cdot \text{HR23}) \]

\[ + (H_2 - K \cdot \text{HR23} / P_1 \cdot P_3) \cdot T_{265} / (\text{UTD} + H_2 + K \cdot \text{HR23}) \]

\[ + P_5 \cdot TA \]

\[ T_2 = -UB / P_1 / (\text{UTD} + H_2 + K \cdot \text{HR23}) \]

\[ + (H_2 - K \cdot \text{HR23} / P_1 \cdot P_3) \cdot T_{265} / (\text{UTD} + H_2 + K \cdot \text{HR23}) \]

\[ + P_5 \cdot TA \cdot IT \cdot \text{TAEF} / P_1 / (\text{UTD} + H_2 + K \cdot \text{HR23}) \]

**C** T3:
\[ (IT \cdot \text{EQ. 0}) \]

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\[ T_3 = -\frac{UB \cdot TAD}{K \cdot HR23/P1} \]
\[ & - \frac{P2 \cdot TA}{Pl} \]
\[ & - \frac{P3 \cdot TFD}{Pl} \]

\[ \text{ELSE} \]
\[ T_3 = -\frac{UB \cdot TAD}{K \cdot HR23/P1} \]
\[ & - \frac{P2 \cdot TA}{Pl} \]
\[ & - \frac{P3 \cdot TFD}{Pl} \]
\[ & - \frac{IT \cdot TAEF}{K \cdot HR23/P1} \]

\[ \text{END IF} \]

\[ c \]
\[ T_1: \]
\[ 555 \]
\[ T_1 = \frac{(LTTD \cdot (T2 - TA) + (H_1 + HR1) \cdot TA)}{(H_1 + HR1 + KCOV1)} \]

\[ c \]
\[ T_4: \]
\[ T_4 = \frac{(K_2 / D_2 \cdot T_3 + (H_4 + HR4) \cdot TAD)}{(H_4 + HR4 + K_2 / D_2)} \]

\[ \text{Calculate utilized heat} \]
\[ \text{IF (M .LE. 0) THEN} \]
\[ QU = 0 \]
\[ \text{ELSE} \]
\[ QU = M \cdot CP \cdot (1 - \exp(-W \cdot L \cdot UL \cdot FD / M / CP)) \cdot (IT \cdot TAEF / UL - TI + TASTAR) \]
\[ QU = M \cdot CP \cdot (TO - TI) \]
\[ \text{END IF} \]

\[ c \]
\[ \text{Check if solutions converge} \]
\[ \text{open (unit=10, file='surf1empl')} \]
\[ \text{write (10,50) TIME,T1,T2,T3,T4,TAEF} \]

\[ \text{C50 format (I1,F9.3,5F8.2)} \]

\[ \text{DIFFER} = 0.05 \]

\[ \text{IF (ABS(T1II-T1) .GT. DIFFER) THEN} \]
\[ \text{GO TO 999} \]
\[ \text{ELSE} \]
\[ \text{IF (ABS(T2II-T2) .GT. DIFFER) THEN} \]
\[ \text{GO TO 999} \]
\[ \text{ELSE} \]
\[ \text{IF (ABS(T3II-T3) .GT. DIFFER) THEN} \]
\[ \text{GO TO 999} \]
\[ \text{ELSE} \]
\[ \text{IF (ABS(T4II-T4) .GT. DIFFER) THEN} \]
\[ \text{GO TO 999} \]
\[ \text{END IF} \]
\[ \text{END IF} \]
\[ \text{END IF} \]
\[ \text{GOTO 300} \]

\[ c \]
\[ \text{Substitute initial guess values as predicted values and continue iteration:} \]

\[ 999 \]

\[ \text{IF (ABS(T1II-T1) .LT. DIFFER) THEN} \]
\[ \text{IF (ABS(T2II-T2) .LT. DIFFER) THEN} \]
\[ \text{IF (ABS(T3II-T3) .LT. DIFFER) THEN} \]
\[ \text{IF (ABS(T4II-T4) .LT. DIFFER) THEN} \]
\[ \text{GOTO 300} \]
\[ \text{END IF} \]
\[ \text{END IF} \]
\[ \text{END IF} \]
\[ \text{END IF} \]
\[ \text{END IF} \]

\[ \text{IF (ABS(T1I-T1) .LT. DIFFER) THEN} \]
\[ \text{IF (ABS(T1I-T1) .LT. DIFFER) THEN} \]

\[ 266 \]
IF (ABS(T1III-T1) LT. DIFFER) THEN
  IF (ABS(T2I5-T2II) LT. DIFFER) THEN
    IF (ABS(T3I4-T3I) LT. DIFFER) THEN
      IF (ABS(T4I5-T4II) LT. DIFFER) THEN
        FLAG=1
        GOTO 300
      END IF
    END IF
  END IF
END IF

TFDI5=TFDI4
T1I5=T1I4
T2I5=T2I4
T3I5=T3I4
T4I5=T4I4

TFDI4=TFDIII
T1I4=T1III
T2I4=T2II
T3I4=T3III
T4I4=T4III

TFDIII=TFDII
T1III=T1III
T2III=T2II
T3III=T3III
T4III=T4III

TFDII=TFDI
T1II=T1II
T2II=T2II
T3II=T3II
T4II=T4II

TFDI=TFD
T1I=T1
T2I=T2
T3I=T3
T4I=T4
GOTO 10

300 OUT(1)=T1
OUT(2)=T2
OUT(3)=T3
OUT(4)=T4
OUT(5)=TFD
OUT(6)=TO
OUT(7)=3.6*UT
OUT(8)=3.6*UE
OUT(9)=3.6*UTD
OUT(10)=3.6*UB
OUT(11)=3.6*KCOVER
OUT(12)=3.6*H1
OUT(13)=3.6*HR1
OUT(14)=3.6*H2
OUT(15)=3.6*H3
OUT(16)=3.6*H4
Appendices

OUT(17)=3.6*HR4
OUT(18)=3.6*QU
OUT(19)=3600*M
OUT(20)=3.6*UL
OUT(21)=FD
OUT(22)=FR
OUT(23)=3.6*HR3S
OUT(24)=TASTAR
OUT(25)=L*W*UTD*(T2-TA)*3.6
OUT(26)=L*W*UB*(T3-TAD)*3.6

C write (10,*), '----------------------------------------------------'
RCVNT 1
END

SUBROUTINE COVER(TLOW, THIGH, ETAC, AR, D, SLOPE, HC, HR)

Subroutine to calculate cover properties

DOUBLE PRECISION TLOW, THIGH, MEANT, KA, VIS, BETA, RA
&, NU1, NU2, NU3, NU4, NU5, GG, NU90, NU60
&, NU, HC, HR, AR

REAL D, ETAC, SLOPE

PARAMETER (PR=0.7, G=9.81, SBC=5.67E-8, RDCONV=0.017453)

C Mean temperature of surface 1 & 2:
MEANT=(TLOW+THIGH)/2

C Conductivity of air in enclosure:
KA=0.000206*(MEANT+273)**0.85

C Viscosity of air in enclosure:
VIS=9E-10*(MEANT+273)**1.72

C Volumetric coefficient of expansion of air:
BETA=I/(MEANT+273)

C Rayleigh number:
RA=G*BETA*ABS(THIGH-TLOW)*D**3*PR/VIS**2

COSSLP=COS(SLOPE*RDCONV)

C Calculate Nusselt number and hence free convective HC12:
IF (SLOPE LE. 75) THEN

NU1=1-1708/(RA*COSSLP)
IF (NU1 .LT. 0) NU1=0

NU2=(RA*COSSLP/6310)**0.3333333-1
IF (NU2 .LT. 0) NU2=0

NU=1+1.44*(1-1708*(SIN(l. 8*RDCONV*SLOPE))**1.6
& /RA/COSSLP)*NU1+NU2

ELSE

NU1=0.0605*RA**(1/3)

NU2=(1+(0.104*RA**0.293/(1+6310/RA)**1.36)**3)**(1/3)

NU3=0.242*(RA/AR)**0.272

NU90=NU1

IF (NU2 .GT. NU90) NU90=NU2

IF (NU3 .GT. NU90) NU90=NU3

GG=0.5/(1+(RA/3160)**20.6)**0.1

NU4=(1+(0.0936*RA**0.314/(1+GG))**7)**(1/7)

NU5=(0.104+0.175/AR)*RA**0.283

NU60=NU4

IF (NU5 .GT. NU0) NU0=NU5

NU=((90-SLOPE)*NU60+(SLOPE-60)*NU90)/30

ENDIF

HC=NU*KA/D
C Radiation heat transfer coefficient between cover outer and inner cover:

\[ \frac{HR=\frac{\sigma \cdot (THIGH-273)^2 \cdot (TLOW+273)^2 \cdot (THIGH-TLOW+546)}}{2/E \cdot AC \cdot (THIGH-TLOW+546)}} \]

RETURN
END
A6.2. Program listing for damper control (TRNSYS)

SUBROUTINE TYP7-87(--Il, l---, X-N, OUT, T, DTDT, PAR, INFO, ICTRL, *)

DOUBLE PRECISION XIN, OUT
DIMENSION XIN(1), PAR(3), OUT(1), INFO(15)

DOUBLE PRECISION R, LMT, THI, TLO, Y, TOD

C INFOs:
INFO(3)=1
INFO(4)=3
INFO(6)=1
CALL TYPECK(1, INFO, INFO(3), INFO(4), 0)

DATA YCHECK /'TH1'/
DATA OCHECK /'CF1'/
CALL RCHECY(Il, --O, YCHECK, OCHECK)

C PARs:
THI=PAR(1)
TLO=PAR(2)
LMT=PAR(3)

C INPUT:
TOD=XIN(1)

C Calculation..
R=1+(TOD-THI)*(1-LMT)/(THI-TLO)
IF (R .GT. 1) THEN
  R=1
END IF
IF (R .LT. LMT) THEN
  R=LMT
END IF
Y=1-R

C OUTPUT:
OUT(1)=Y
RETURN 1
END
Appendices

A6.3. Program listing for air heater battery used in this study (TRNSYS)

SUBROUTINE TYPE73 (TIME, XIN, OUT, DT, PAR, INFO, ICNTRL, *)

C***********************************************************************
C It is a heater that delivered Y*QMAX amount of heat to the fluid.
C This component can be used where
C - source of heating is unknown, and
C - the only concern is to find the energy required
C
C Modified: 15th Mar 1997 from TYPE 6
C***********************************************************************
C PARs:
C QMAX = maximum heating rate delivered to fluid (W)
C CP = specific heat capacity of fluid (J/(kg*C))
C UA = overall loss coefficient between heater and its surrounding during operation (W/°C)
C HTREFF = heater efficiency (-)
C
C INPUTS:
C TIN = inlet temperature to heater (°C)
C FLOW = mass flow rate of fluid (kg/s)
C Y = control function (-)
C TAMB = surrounding environment temperature (°C)
C
C OUTPUTs:
C TOUT = outlet temperature from heater (°C)
C FLOW = mass flow rate of fluid (kg/s)
C QAUX = required heating rate from source [eg. boiler] (W)
C QLOSS = heat loss between heater and its surrounding (W)
C QFLUID = heat delivered to fluid (W)
C***********************************************************************

DOUBLE PRECISION XIN, OUT
INTEGER*4 INFO
DIMENSION XIN(4), OUT(5), PAR(4), INFO(15)
CHARACTER*3 YCHECK(4), OCHECK(5)
REAL TIN, TOUT, TEAR, TAMB
REAL QMAX, QAUX, QLCSS, QFLUID
REAL FLOW, CP, HTREFF, Y

IF (INFO(7).GE.0) GO TO 100
C FIRST CALL OF SIMULATION
INFO(6)=5
INFO(9)=0
CALL TYPECK(1, INFO, 4, 4, 0)

DATA YCHECK/'TEI', 'MF1', 'CF1', 'TE1'/
DATA OCHECK/'TE1', 'MF1', 'PW1', 'PW1', 'PW1'/
CALL RCHECK(INFO, YCHECK, OCHECK)

C SET PARAMETER AND INPUT VARIABLES
100 QMAX=PAR(1)
CP=PAR(2)
UA=PAR(3)
HTREFF=PAR(4)
TIN=XIN(1)
FLOW=XIN(2)
Y=XIN(3)
TAMB=XIN(4)

IF (FLOW .GT. 0.) GO TO 10
C NO FLOW
  OUT(1) = TIN
  OUT(2) = 0.
  OUT(3) = 0.
  OUT(4) = 0.
  OUT(5) = 0.
  RETURN 1

C HEATER ON
10 CONTINUE
  QFLUID = Y*QMAX
  TOUT = TIN + QFLUID/FLOW/CP
  TBAR = (TIN + TOUT)/2.
  QLOSS = UA*(TBAR - TAMB) + (1.-HTREFF)*Y*QMAX
  QAUX = QLOSS + QFLUID

C SET OUTPUTS
50 OUT(1) = TOUT
  OUT(2) = FLOW
  OUT(3) = QAUX
  OUT(4) = QLOSS
  OUT(5) = QFLUID

RETURN 1
END
A6.4. Program listing for estimating heat loss due to axial conduction

A6.4.1. Estimated loss from absorber by axial conduction

```
PROGRAM EZTE-1--D

This program is designed to find out the extra heat loss by conduction through extension.

DOUBLE PRECISION V, H3, HR3, H3D, K2, D2
DOUBLE PRECISION TFF, T4
DOUBLE PRECISION QEXT, KSTEEL, AXPROF, R1, TTIP, C3, TB, L1

PARAMETER (SBC=5.67E-8)

KSTEEL=50
AXPROF=1.168E-3

PRINT *, 'H3: '
READ *, H3

PRINT *, 'K2: '
READ *, K2

PRINT *, 'D2: '
READ *, D2

PRINT *, 'TFF (fluid temperature): '
READ *, TFF

PRINT *, 'TTIP (temperature at the tip): '
READ *, TTIP

PRINT *, 'TB (base temperature): '
READ *, TB

PRINT *, 'T4: '
READ *, T4

PRINT *, 'W: '
READ *, W

L1=0.4

H3D=H3+HR3
PRINT *, 'H3D= ', H3D

R1=SQRT(W/KSTEEL*AXPROF*(H3D+K2/D2))
PRINT *, 'R1= ', R1

C3=(H3D*TFF+K2*D2*T4)/(H3D+K2/D2)
PRINT *, 'C3= ', C3

QEXT=KSTEEL*AXPROF*R1*{(TTIP-C3)-(TB-C3)*COSH(R1*L1)} & /
    SINH(R1*L1)
```

273
PRINT *, 'QEXT= ', QEXT

END
A6.4.2. Estimated loss from edge to ambient environment

PROGRAM EDGE

DOUBLE PRECISION L, KSTEEL, AXPROF, UE, H2, DF, KF, VPF, ETAE, ETAF, TA, TE & TENV, TTIPD, TTIPDD, HR2, H2D, UF, HF, HRF, HFD, R4, R5, R6 & C7, C8, C9, C10, C11, C12, C13, C14, C15, C16, C17, C18, TE1, TE2, L2, L3, L4 & QLOSS, QLOSSI, QLOSS2

PARAMETER (SBC=5.67E-8)

L=1
L2=0.13
L3=0.045
L4=0.177
KSTEEL=50
AXPROF=0.001

PRINT *, 'UE: '
READ *, UE

PRINT *, 'H2: '
READ *, H2

DF=0.025
KF=200

PRINT *, 'VPF: '
READ *, VPF

ETAE=0.96
ETAF=0.7

PRINT *, 'TA: '
READ *, TA

PRINT *, 'TE: '
READ *, TE

PRINT *, 'TENV (2/3*MRT+1/3*TA): '
READ *, TENV

PRINT *, 'TTIPD: '
READ *, TTIPD

PRINT *, 'TTIPDD: '
READ *, TTIPDD

HR2=ETAE*SBC*((TE+273)**2+(TENV+273)**2)*(TE+TENV+546)
PRINT *, 'HR2=', HR2

H2D=H2+HR2
PRINT *, 'H2D=', H2D

UF=1/(0.06+DF/KF)
PRINT *, 'UF=', UF

HF=5.7+3.8*VPF
PRINT *, 'HF=', HF

HRF = ETAF * SBC * ((TA + 273) ** 2 + ((TE + TA) / 2 + 273) ** 2) * ((TE + TA) / 2 + TA + 546)
PRINT *, 'HRF=', HRF

HFD = HF + HRF
PRINT *, 'HFD=', HFD

R4 = SQRT (L / KSTEEL / AXPROF * (UE + H2D))
PRINT *, 'R4=', R4

R5 = SQRT (L * (UF + HFD) / KSTEEL / AXPROF)
PRINT *, 'R5=', R5

R6 = SQRT (2 * HFD / L / KSTEEL / AXPROF)
PRINT *, 'R6=', R6

C9 = (UE + TA + H2D + TENV) / (UE + H2D)
PRINT *, 'C9=', C9

C14 = TTIPDD - C9
PRINT *, 'C14=', C14

C15 = (TE - C9 - (TTIPDD - C9) * COSH (R4 * L2 / 2)) / SINH (R4 * L2 / 2)
PRINT *, 'C15=', C15

C16 = R4 * COSH (R4 * L2 / 2) / SINH (R4 * L2 / 2) + R5 * COSH (R5 * L3) / SINH (R5 * L3)
PRINT *, 'C16=', C16

C17 = R5 * COSH (R5 * L3) / SINH (R5 * L3) + R6 * COSH (R6 * L4) / SINH (R6 * L4)
PRINT *, 'C17=', C17

C18 = 1 - R5 ** 2 / C16 / C17 / (SINH (R5 * L3)) ** 2
PRINT *, 'C18=', C18

TE1 = R4 / C16 / C18 / SINH (R4 * L2 / 2) * TE
& + R5 / C16 / C17 / C18 / SINH (R5 * L3) / SINH (R6 * L4) * TTIPDD
& + R5 / C16 / C18 / SINH (R5 * L3)
& * (1 - COSH (R5 * L3)) + R5 / C17 / SINH (R5 * L3) * (1 - COSH (R5 * L3))
& + R6 / C17 / SINH (R6 * L4) * (1 - COSH (R6 * L4)) * TA
PRINT *, 'TE1=', TE1

TE2 = R5 / C17 / SINH (R5 * L3) * TE1
& + R6 / C17 / SINH (R6 * L4) * TTIPDD
& - (R5 / C17 / SINH (R5 * L3)) * (1 - COSH (R5 * L3))
& + R6 / C17 / SINH (R6 * L4) * (1 - COSH (R6 * L4)) * TA
PRINT *, 'TE2=', TE2

C12 = SINH (R6 * (L2 + L3 + L4)) / SINH (R6 * L4) * TE2
& - SINH (R6 * (L2 + L3)) / SINH (R6 * L4) * TTIPDD
& + (SINH (R6 * (L2 + L3)) - SINH (R6 * (L2 + L3 + L4))) / SINH (R6 * L4) * TA
PRINT *, 'C12=', C12

C13 = COSH (R6 * (L2 + L3 + L4)) / SINH (R6 * L4) * TE2
& + COSH (R6 * (L2 + L3)) / SINH (R6 * L4) * TTIPDD
& - (COSH (R6 * (L2 + L3)) - COSH (R6 * (L2 + L3 + L4))) / SINH (R6 * L4) * TA
PRINT *, 'C13=', C13

C10 = SINH (R5 * (L2 + L3)) / SINH (R5 * L3) * TE1
& - SINH (R3 * L2) / SINH (R5 * L3) * TE2
& + (SINH (R5 * L2) - SINH (R5 * (L2 + L3))) / SINH (R5 * L3) * TA
PRINT *, 'C10=', C10

C11 = - COSH (R3 * (L2 + L3)) / SINH (R5 * L3) * TE1
& + COSH (R5 * L2) / SINH (R5 * L3) * TE2
\begin{verbatim}
4 - (COSH(R5*L2) - COSH(R5*(L2+L3)))/SINH(R5*L3) * TA
PRINT *, 'C11=', C11

C7 = SINH(R4*L2)/SINH(R4*L2/2) * T - T1
4 - (SINH(R4*L2) - SINH(R4*L2/2))/SINH(R4*L2/2) * C9
PRINT *, 'C7=', C7

C8 = -COSH(R4*L2)/SINH(R4*L2/2) * T + COSH(R4*L2/2)/SINH(R4*L2/2) * T1
& (COSH(R4*L2) - COSH(R4*L2/2))/SINH(R4*L2/2) * C9
PRINT *, 'C8=', C8

QLOSS1 = R4*KSTEEL*A. XPROF*C15
PRINT *, 'QLOSS1=', QLOSS1

QLOSS2 = -R4*KSTEEL*A. XPROF*(C7*SINH(R4*L2) + C8*COSH(R4*L2))
PRINT *, 'QLOSS2=', QLOSS2

QLOSS = QLOSS1 + QLOSS2
PRINT *, 'QLOSS=', QLOSS

END
\end{verbatim}
A7. Miscellaneous equations for the proposed mathematical model

The following equations are to be used in conjunction with those presented in Chapter 2.

\[ P_1 = \frac{K_{hr_{23}}}{U_i' + h_2 + K_{hr_{23}}} - \frac{U_b}{K_{hr_{23}}} - \frac{h_{r_{23}}}{h_{r_{23}}} - \frac{h_3}{h_{r_{23}}} - 1 \] ... (A7.1)

\[ P_2 = \frac{U_i'}{U_i' + h_2 + K_{hr_{23}}} + \frac{h_{r_{23}}}{h_{r_{23}}} \] ... (A7.2)

\[ P_3 = \frac{h_3}{h_{r_{23}}} + \frac{h_2}{U_i' + h_2 + K_{hr_{23}}} \] ... (A7.3)

\[ P_4 = \frac{h_2}{U_i' + h_2 + K_{hr_{23}}} - \frac{K_{hr_{23}} P_3}{P_1 (U_i' + h_2 + K_{hr_{23}})} - 1 \]
\[ = \frac{1}{U_i' + h_2 + K_{hr_{23}}} \left[ h_2 - \frac{K_{hr_{23}} P_3}{P_1} \right] - 1 \] ... (A7.4)

\[ P_5 = \frac{U_i'}{U_i' + h_2 + K_{hr_{23}}} - \frac{K_{hr_{23}} P_2}{P_1 (U_i' + h_2 + K_{hr_{23}})} \]
\[ = \frac{1}{U_i' + h_2 + K_{hr_{23}}} \left[ U_i' - \frac{K_{hr_{23}} P_2}{P_1} \right] \] ... (A7.5)

\[ P_6 = h_2 P_4 - K_{hr_3} \left( \frac{P_3}{P_1} + 1 \right) \] ... (A7.6)

\[ P_7 = h_2 P_5 - \frac{K_{hr_3} P_2}{P_1} \] ... (A7.7)
Mathematical model describing the thermal performance for no-flow situation in the proposed solar collector system was derived as follow:

\[ K_{\text{cover}} d^2 \]

Figure A7.1: Heat transfer mechanism of the collector in a no-flow situation

**Absorber**

\[ U_h(T_3 - t_{a\prime}) + (h_{23} + Kh_{r23})(T_3 - T_2) = I(\tau \alpha) \]

... (A8.1)

**Cover**

\[ U_i'(T_2 - t_a) = (h_{23} + Kh_{r23})(T_3 - T_2) \]

... (A8.2)

**Fluid**

\[ \bar{t}_f = \frac{1}{2}(T_2 + T_3) \]

... (A8.3)

Solving Equation A8.1 for \( T_2 \) yields
\[ T_2 = \frac{(U_h + h_{23} + Kh_{23})T_3 - U_h t_a - I(t\alpha)_e}{h_{23} + Kh_{23}} \] ...

(A8.4)

Substituting Equation A8.4 into Equation A8.2 yields

\[ T_3 = \frac{U_t (U_t + h_{23} + Kh_{23}) t_a + (U_t + h_{23} + Kh_{23}) I(t\alpha)_e + (h_{23} + Kh_{23}) t_a}{U_t (U_t + h_{23} + Kh_{23}) + U_t (h_{23} + Kh_{23})} \] ...

(A8.5)
Appendices

A9. Estimation of boiler capacity from individual zone load

1. Weight training room

Room Dimensions : 12m x 8m x 3.2m
Extra Areas : None
Perimeter : External Wall 17.6m, Internal Wall 22.4m
Room Temperature : 16.0 deg C
Above Temperature : -1.0 deg C
Structure Type
External Wall : (User defined)
Internal Walls (1)
Int. Wall 1 : (User defined)
Roof : (User defined)
Ground Floor : (User defined)
Air Changes : 1 per hour
Windows (2)
Window 1 : (User defined)
Window 2 : (User defined)
Heatloss : 4354 Watts
Selected : No Selection
Output Required : 4354W

2. Sport Hall

Room Dimensions : 32m x 17m x 7.6m
Extra Areas : None
Perimeter : External Wall 66m, Internal Wall 32m
Room Temperature : 16.0 deg C
Above Temperature : -1.0 deg C
Structure Type
External Wall : (User defined)
Internal Walls (1)
Int. Wall 1 : (User defined)
Roof : (User defined)
Ground Floor : (User defined)
Air Changes : 1 per hour
Windows (2)
Window 1 : (User defined)
Window 2 : (User defined)
Heatloss : 46534 Watts
Selected : No Selection
Output Required : 46534W
Appendices

3. General office

Room Dimensions: 6.8m x 3.8m x 3.2m  
Extra Areas: None  
Perimeter: External Wall 6.8m, Internal Wall 14.4m  
Room Temperature: 21.0 deg C  
Above Temperature: 21.0 deg C  
Structure Type  
External Wall: (User defined)  
Internal Walls: 2  
Int. Wall 1: (User defined)  
Ceiling: (User defined)  
Ground Floor: (User defined)  
Air Changes: 1 per hour  
Windows: 1  

density  
Heatloss: 1415 Watts  
Selected: No Selection  

4. Bar & lounge

Room Dimensions: 18m x 12m x 3.2m  
Extra Areas: None  
Perimeter: External Wall 12m, Internal Wall 48m  
Room Temperature: 21.0 deg C  
Adjacent Temperature: 16.0 deg C  
Below Temperature: -1.0 deg C  
Structure Type  
External Wall: (User defined)  
Internal Walls: 3  
Int. Wall 1: (User defined)  
Roof: (User defined)  
Ground Floor: (User defined)  
Air Changes: 1 per hour  
Windows: 1  

Heatloss: 10422 Watts  
Selected: No Selection  

Output Required: 1582W  
Supplied: 0W  
Variance: -100.0%  
Total Cost: £0.00
Appendices

5. Store

Room Dimensions: 16m x 3.84m x 3.2m
Extra Areas: None
Perimeter: External Wall 0m, Internal Wall 39.68m
Room Temperature: 16.0 deg C
Above Temperature: 21.0 deg C
Adjacent Temperature: 16.0 deg C
Below Temperature: -1.0 deg C

<table>
<thead>
<tr>
<th>Structure</th>
<th>Type</th>
<th>U-Value</th>
<th>Area/Vol</th>
<th>Heatloss</th>
<th>% of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall</td>
<td>(User defined)</td>
<td>0.37</td>
<td>0.0 sq.m</td>
<td>0W</td>
<td>0.0</td>
</tr>
<tr>
<td>Internal Walls (1)</td>
<td></td>
<td></td>
<td>0.0 sq.m</td>
<td>0W</td>
<td>0.0</td>
</tr>
<tr>
<td>Int. Wall 1</td>
<td>(User defined)</td>
<td>0.00</td>
<td>39.68 m x 3.2 m</td>
<td>0W</td>
<td>0.0</td>
</tr>
<tr>
<td>Ceiling</td>
<td>(User defined)</td>
<td>0.00</td>
<td>61.4 sq.m</td>
<td>0W</td>
<td>0.0</td>
</tr>
<tr>
<td>Ground Floor</td>
<td>(User defined)</td>
<td>0.39</td>
<td>61.4 sq.m</td>
<td>407W</td>
<td>23.1</td>
</tr>
<tr>
<td>Air Changes</td>
<td>1 per hour</td>
<td></td>
<td>196.6 cu.m</td>
<td>1136W</td>
<td>76.9</td>
</tr>
<tr>
<td>Windows (0)</td>
<td></td>
<td></td>
<td>0.0 sq.m</td>
<td>0W</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Heatloss: 1763 Watts
Selected: No Selection
N/A       100.0%   N/A
Total Cost: $0.00
Output Required: 1763W
Supplied: 0W Variance -100.0%

6. Circulation area

Room Dimensions: 22m x 6m x 3.2m
Extra Areas: None
Perimeter: External Wall 3.2m, Internal Wall 52.8m
Room Temperature: 21.0 deg C
Above Temperature: 21.0 deg C
Adjacent Temperature: 21.0 deg C
Below Temperature: -1.0 deg C

<table>
<thead>
<tr>
<th>Structure</th>
<th>Type</th>
<th>U-Value</th>
<th>Area/Vol</th>
<th>Heatloss</th>
<th>% of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall</td>
<td>(User defined)</td>
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<td>8.0 sq.m</td>
<td>75W</td>
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<tr>
<td>Internal Walls (1)</td>
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<td></td>
<td>0.0 sq.m</td>
<td>0W</td>
<td>0.0</td>
</tr>
<tr>
<td>Int. Wall 1</td>
<td>(User defined)</td>
<td>0.00</td>
<td>52.8 m x 3.2 m</td>
<td>0W</td>
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<tr>
<td>Ceiling</td>
<td>(User defined)</td>
<td>0.00</td>
<td>132.0 sq.m</td>
<td>0W</td>
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<tr>
<td>Ground Floor</td>
<td>(User defined)</td>
<td>0.39</td>
<td>132.0 sq.m</td>
<td>1132W</td>
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<td>Air Changes</td>
<td>1 per hour</td>
<td></td>
<td>422.4 cu.m</td>
<td>3643W</td>
<td>72.7</td>
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<tr>
<td>Windows (1)</td>
<td></td>
<td></td>
<td>2.2 sq.m</td>
<td>162W</td>
<td>3.2</td>
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</tbody>
</table>

Window 1: (User defined)

Heatloss: 5012 Watts
Selected: No Selection
N/A       100.0%   N/A
Total Cost: $0.00
Output Required: 5604W
Supplied: 0W Variance -100.0%
### 7. WCS

**Room Dimensions:** 9m x 4.3m x 3.2m  
**Room Level:** Bungalow  
**Structure Type:**  
- External Wall (User defined)  
- Internal Walls (1)  
- Int. Wall 1: (User defined)  
- Roof (User defined)  
- Ground Floor: (User defined)  
**Air Changes:** 1 per hour  
**Windows:** None  
**Heat Loss:** 1715 Watts  
**Selected:** No Selection  
**Output Required:** 1917W

<table>
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<th>Structure Type</th>
<th>U-Value</th>
<th>Area/Vol</th>
<th>Heat Loss % of total</th>
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</thead>
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<tr>
<td>External Wall</td>
<td>0.37</td>
<td>0.0 sq.m</td>
<td>0W 0.0</td>
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<tr>
<td>Internal Walls</td>
<td>0.00</td>
<td>26.6 m x 3.2 m</td>
<td>0W 0.0</td>
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<td>Roof</td>
<td>0.37</td>
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<td>315W 18.4</td>
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<td>Ground Floor</td>
<td>0.39</td>
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<td>332W 19.4</td>
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<td>123.8 cu.m</td>
<td>1068W 62.3</td>
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<td>Windows</td>
<td>0.00</td>
<td>0.0 sq.m</td>
<td>0W 0.0</td>
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</tbody>
</table>

**Selected No Selection**  
**Output Required:** 1917W  
**Supplied:** 0W  
**Variance:** -100.0%

### 8. Changing Room

**Room Dimensions:** 12m x 12m x 3.2m  
**Room Level:** Bungalow  
**Structure Type:**  
- External Wall (User defined)  
- Internal Walls (1)  
- Int. Wall 1: (User defined)  
- Roof (User defined)  
- Ground Floor: (User defined)  
**Air Changes:** 4 per hour  
**Windows:** 2  
**Selected:** No Selection  
**Output Required:** 29271W

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<th>Structure Type</th>
<th>U-Value</th>
<th>Area/Vol</th>
<th>Heat Loss % of total</th>
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<td>External Wall</td>
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<td>12.8 sq.m</td>
<td>140W 0.6</td>
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<tr>
<td>Internal Walls</td>
<td>0.00</td>
<td>36 m x 3.2 m</td>
<td>0W 0.0</td>
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<td>Roof</td>
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<td>144.0 sq.m</td>
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<td>Ground Floor</td>
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<td>144.0 sq.m</td>
<td>1628W 6.9</td>
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<tr>
<td>Air Changes</td>
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<td>460.8 cu.m</td>
<td>18444W 77.6</td>
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<td>Windows</td>
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<td>25.6 sq.m</td>
<td>2158W 9.1</td>
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<td>Window 1</td>
<td>2.91</td>
<td>7.68 m x 1 m</td>
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<tr>
<td>Window 2</td>
<td>2.91</td>
<td>17.9 m x 1 m</td>
<td></td>
</tr>
</tbody>
</table>

**Heat Loss:** 23755 Watts  
**Selected:** No Selection  
**Output Required:** 29271W  
**Supplied:** 0W  
**Variance:** -100.0%
Appendices

9. Staff room

Project Title: Sash model01

Room Dimensions: 4m x 2.5m x 3.2m
Extra Areas: None
Perimeter: External Wall 2.5m, Internal Wall 10.5m

Room Temperature: 21.0 deg C
Above Temperature: -1.0 deg C

Structure Type

External Wall: (User defined) U-Value 0.37 Area/Volume Heatloss % of total
Internal Walls (1): 0.00 10.5 m x 3.2 m (21 deg. C)
Int. Wall 1: (User defined) 0.37 10.0 sq.m 81W 12.9
Roof: (User defined) 0.37 10.0 sq.m 81W 12.9
Ground Floor: (User defined) 0.39 10.0 sq.m 97W 15.4
Air Changes: 1 per hour 32.0 cu.m 276W 43.8
Windows (1): 1.6 sq.m 116W 18.4
Window 1: (User defined) 2.91 1.6 m x 1 m

Heatloss: 630 Watts
Selected: No Selection

Output Required: 704W
Supplied: 0W
Variance: -100.0%

10. First aid

Project Title: Sash model01

Room Dimensions: 4m x 2.4m x 3.2m
Extra Areas: None
Perimeter: External Wall 2.4m, Internal Wall 10.4m

Room Temperature: 21.0 deg C
Above Temperature: -1.0 deg C

Structure Type

External Wall: (User defined) U-Value 0.37 Area/Volume Heatloss % of total
Internal Walls (1): 0.00 10.4 m x 3.2 m (21 deg. C)
Int. Wall 1: (User defined) 0.37 10.0 sq.m 81W 12.9
Roof: (User defined) 0.37 9.6 sq.m 78W 13.1
Ground Floor: (User defined) 0.39 9.6 sq.m 82W 13.8
Air Changes: 1 per hour 30.7 cu.m 265W 44.6
Windows (1): 1.5 sq.m 112W 18.9
Window 1: (User defined) 2.91 1.54 m x 1 m

Heatloss: 594 Watts
Selected: No Selection

Output Required: 664W
Supplied: 0W
Variance: -100.0%

Total Cost: £0.00

Supplied: 0W
Variance: -100.0%
A10. Estimation of heat gain by lighting

The following equations were used for evaluation of lighting gain in Chapter 5:

\[
\text{Installed flux} = \frac{\text{Illuminance} \times \text{area}}{\text{UF} \times \text{LLF}}
\]

\[
\text{Number of lamps required} = \frac{\text{Installed flux}}{\text{Output of lamp}}
\]

So combine the above two equations, the following was obtained:

\[
\text{Number of lamps required} = \frac{\text{Illuminance} \times \text{area}}{\text{UF} \times \text{LLF} \times \text{output of lamp}}
\]

where

- \( k \) Room index
  \[ (= \frac{LW}{H_m(L + W)}) \]
- \( \text{LLF} \) Light loss factor (taken as that for fluorescent tube in an enclosed diffuser for office use \([0.74]\))
- \( \text{UF} \) Utilization factor
A11. Extra pressure loss due to extra ductwork

Pressure drop across collector channel was derived using the following equations:

\[
\frac{1}{\sqrt{f}} = -4 \log_{10} \left[ \frac{e}{3.7d_h} + \frac{1.255}{\text{Re} \sqrt{f}} \right] \quad \text{(A11.1)}
\]

\[
\Delta p = 4f \left( \frac{1}{d_h} \right) \left( \frac{\rho v^2}{2} \right) \quad \text{(A11.2)}
\]

where

- \(d_h\) = hydraulic diameter of collector channel (m)
- \(e\) = surface roughness of collector channel (m)
- \(f\) = friction (-)
- \(\text{Re}\) = Reynolds number of fluid in collector channel (-)
- \(v\) = velocity of fluid in collector channel (ms\(^{-1}\))
- \(\rho\) = density of fluid in channel (kgm\(^{-3}\))
- \(\Delta p\) = pressure drop across collector channel (Pa)

The relationship between pressure loss and collector width is shown in Figure A11.1.

Figure A11.1: Graph showing the relationship between collector width against pressure loss across the collector channel for different collector geometries
The pressure drop across the appropriate ducting system, as illustrated in Figure A11.3, was estimated using HEVACOMP. The following graphs shows the results:

**Pressure loss due to extra ductwork**

(velocity $< 2 \text{ m/s}$, $\Delta p < 0.1 \text{ Pa m}^{-1}$)

![Graph showing pressure loss across ductwork system for different collector width](image)

**Figure A11.2:** Pressure loss across ductwork system for different collector width (velocity limit of $2 \text{ m/s}$ and pressure drop limit of $0.1 \text{ Pa m}^{-1}$ was used)

**Figure A11.3:** Schematic of the associated ductwork
The following listing is the results of duct sizing using the HEVACOMP package:

<table>
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<th>Section no.</th>
<th>Flow rate (m3/s)</th>
<th>Duct size (mm)</th>
<th>Type</th>
<th>Velocity (m/s)</th>
<th>Section loss (N/m²)</th>
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Fan total pressure: 9 N/m²
Fan static pressure: 7 N/m²
Fan volume: 2.10 m³/s
Index run to section 60
(sections 60, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1)

--- Damper schedule ---

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### Fittings schedule

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### Terminators schedule

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### Inlets schedule

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Duct System weight

Total system weight 4868 kg
(force 47754 N)

Total surface area 625.6 m2